Exclusive meson production as a probe of parton orbital dynamics

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Based on: PRL 133, 051901 (24) & 2601.17506 (26)

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Duxin Zheng (Shandong U)

Jian Zhou (Shandong U)

Outline

- Generalized TMDs & connection to spin physics
- Observable(s) for quark/gluon OAM & spin-orbit correlations:
  1) Exclusive pseudoscalar meson &
  2) Exclusive heavy (axial-) vector meson production
- Summary

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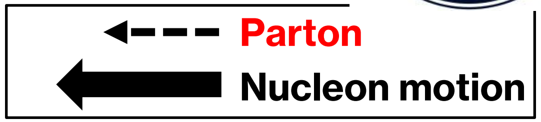
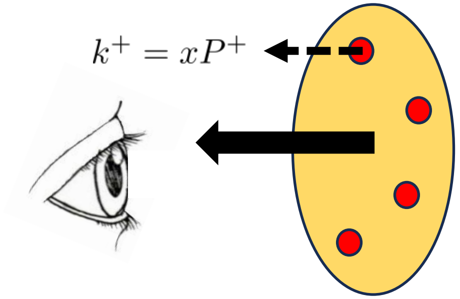
GTMDs - The “mother function”

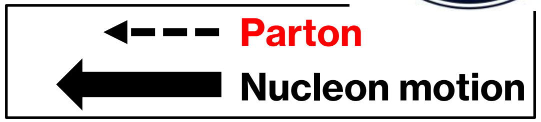


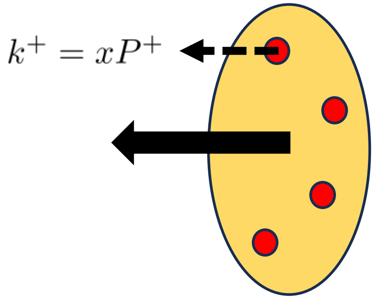


Parton Distribution Functions

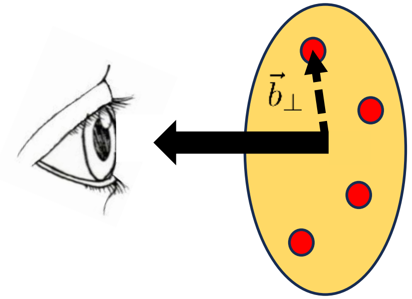
PDFs (x)

GTMDs - The “mother function”





Form Factors



PDFs (x)

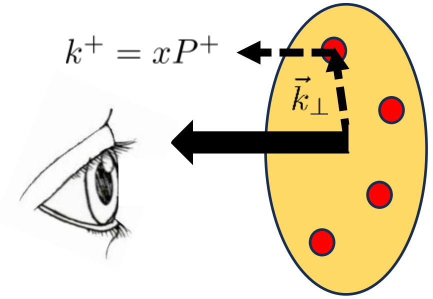
FFs (Δ)

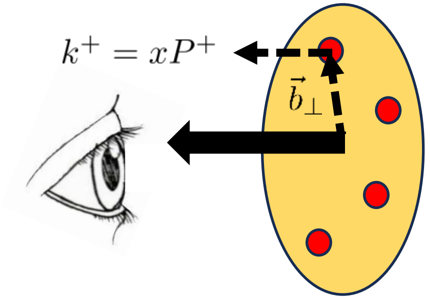
GTMDs - The "mother function"

Parton
Nucleon motion

Transverse Momentum-dependent Distributions

Generalized Parton Distributions





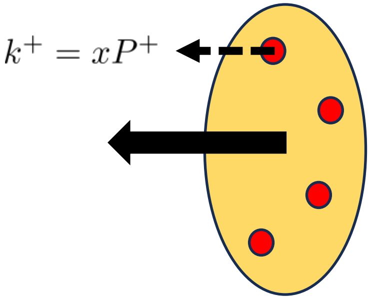
TMDs $(x, \vec{k}_\perp)$

GPDs (x, Δ)

$\int d^2\vec{k}_\perp$

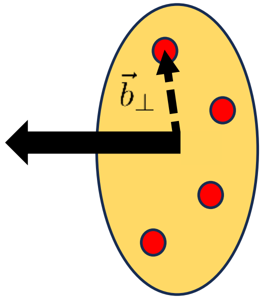
$\Delta = 0$

$\int dx$



PDFs (x)

FFs (Δ)



GTMDs - The “mother function”

Generalized Transverse Momentum-dependent Distributions

(Meissner, Metz, Schlegel, 2009)

$$
\begin{tikzcd}
 & \textbf{GTMDs}\ (x, \vec{k}_\perp, \Delta) \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int d^2\vec{k}_\perp"] & & \\
\textbf{TMDs}\ (x, \vec{k}_\perp) \arrow[dr, "\int d^2\vec{k}_\perp"'] & & \textbf{GPDs}\ (x, \Delta) \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int dx"] & \\
 & \textbf{PDFs}\ (x) & & \textbf{FFs}\ (\Delta)
\end{tikzcd}
$$

GTMDs/Wigner functions - The “mother function”

Wigner functions $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)

2-D Fourier Transform

$(\vec{\Delta}_\perp)$

(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

$\int d^2\vec{k}_\perp$

TMDs $(x, \vec{k}_\perp)$

GPDs (x, Δ)

$\int d^2\vec{k}_\perp$

$\Delta = 0$

$\int dx$

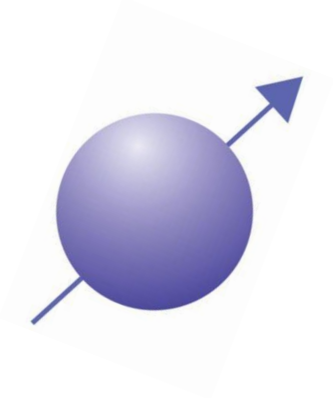
PDFs (x)

FFs (Δ)

Spin of proton

Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2}$$

Spin of proton

Jaffe-Manohar spin decomposition

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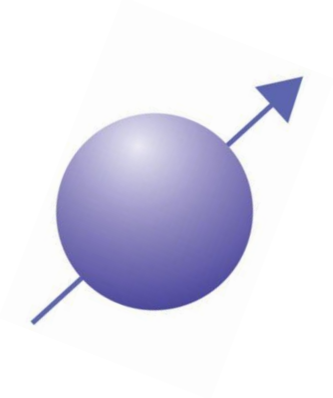

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma$$

Best known

Quark helicity $\sim 30\%$

Spin of proton

Jaffe-Manohar spin decomposition

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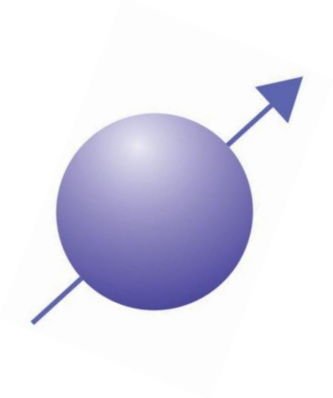


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G$$

Best known

How well do we know?

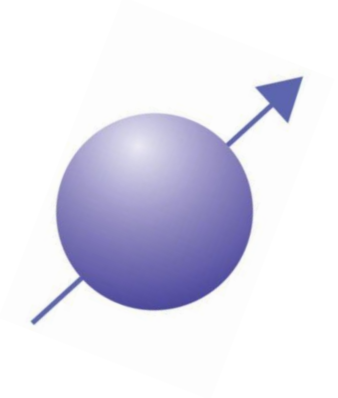
Quark helicity $\sim 30\%$

Gluon helicity $\sim 40\%$

Spin of proton

Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Best known **How well do we know?** **???????**

Quark helicity $\sim 30\%$ **Gluon helicity** $\sim 40\%$ **OAM of quarks & gluons**

So far, <u>no</u> experimental constraints on OAM of quarks & gluons

Wigner functions & Orbital Angular Momentum

Wigner functions in Quantum Mechanics

(Wigner, 1932)

- Calculate from wave functions:

$$W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'}\,\psi(x+\frac{x'}{2})\,\psi^*(x-\frac{x'}{2})$$

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$$

Wigner functions & Orbital Angular Momentum

Wigner functions in Quantum Mechanics (Wigner, 1932)	Wigner functions in parton physics (Belitsky, Ji, Yuan, 2003)
• Calculate from wave functions: $W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x+\frac{x'}{2})\, \psi^*(x-\frac{x'}{2})$	• Calculate from fourier transform of GTMD correlator: $W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$
• Expectation value of observables: $\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$	• Application: **O**rbital **A**ngular **M**omentum (**OAM**) $L_z^{q,g} = \int dx \int d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z\, W^{q,g}\ (x, \vec{b}_\perp, \vec{k}_\perp)$ (Lorcé, Pasquini, 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum

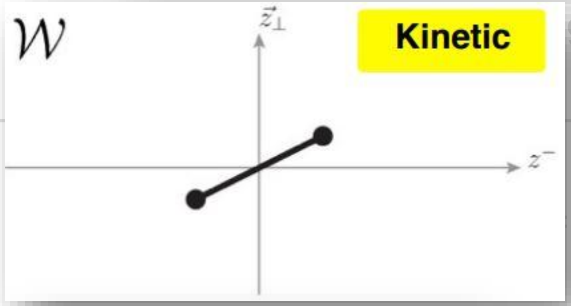
Wigner functions in Quantum Mechanics (Wigner, 1932)	Wigner functions in parton physics (Belitsky, Ji, Yuan, 2003)
• Calculate from wave functions: $W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x+\frac{x'}{2})\, \psi^*(x-\frac{x'}{2})$	• Calculate from fourier transform of GTMD correlator: $W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$
• Expectation value of observables: $\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$	• Application: Relation between GTMD $F_{1,4}^{q,g}$ & OAM

$$\boxed{L_z^{q,g} = -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, k_\perp, \xi = 0, \Delta_\perp = 0)}$$

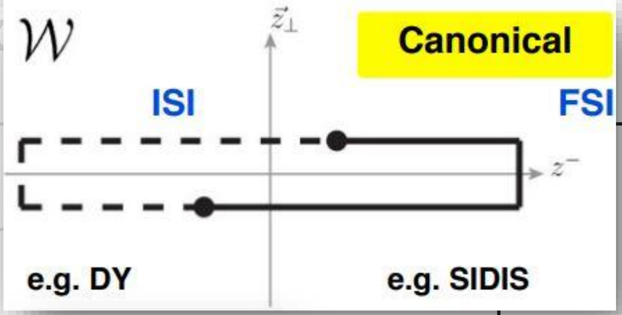
(Lorcé, Pasquini, 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum

Gauge-invariant extension



Ji, Xiong, Yuan, 2012



Hatta, 2011

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$$

Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

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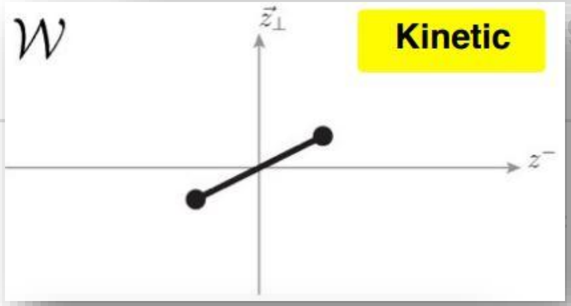

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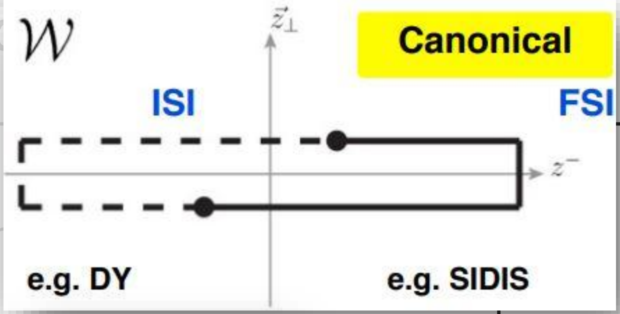


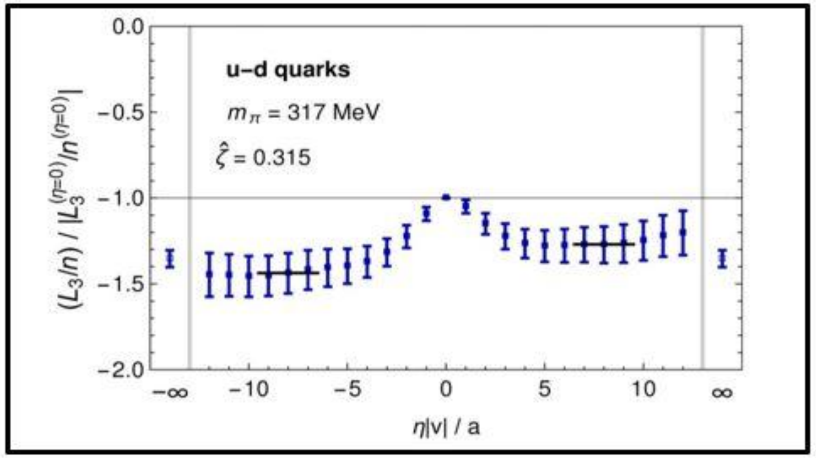
Hatta, 2011

Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

Calculate from fourier transform of GTMD correlator:

$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$



First lattice calculation of $L_{\rm JM}$ vs. $L_{\rm Ji}$
(Engelhardt, 1701.01536)

i. **Figure shows** $L_{\rm JM}^{u-d}/L_{\rm Ji}^{u-d}$

ii. **Significant numerical differences between** $L_{\rm JM}$ **&** $L_{\rm Ji}$

...MD $F_{1,4}^{q,g}$ & OAM

$\ldots, \xi = 0, \Delta_\perp = 0)$

..., 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum

Wigner functions in Quantum Mechanics

(Wigner, 1932)

- Expe

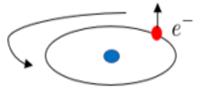
$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$$

Spin-orbit entanglement in the Color Glass Condensate

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[3,4,‡]

2404.04208

Recall Spin-Orbit coupling in H atom!



$$\boxed{G_{1,1}^{q/g} \to L^{q/g} \cdot S^{q/g}}$$

Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: Relation between GTMD $G_{1,1}^{q,g}$ & spin-orbit correlations

$$\boxed{C^{q,g} = \int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} G_{1,1}^{q,g}(x, k_\perp, \xi = 0, \Delta_\perp = 0)}$$

(Lorcé, Pasquini, 2011 / SB, Boussarie, Hatta, 2024)

GTMDs & Orbital Angular Momentum

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f GTMD correlator:

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(Lorcé, Pasquini, 2011 /
SB, Boussarie, Hatta, 2024)

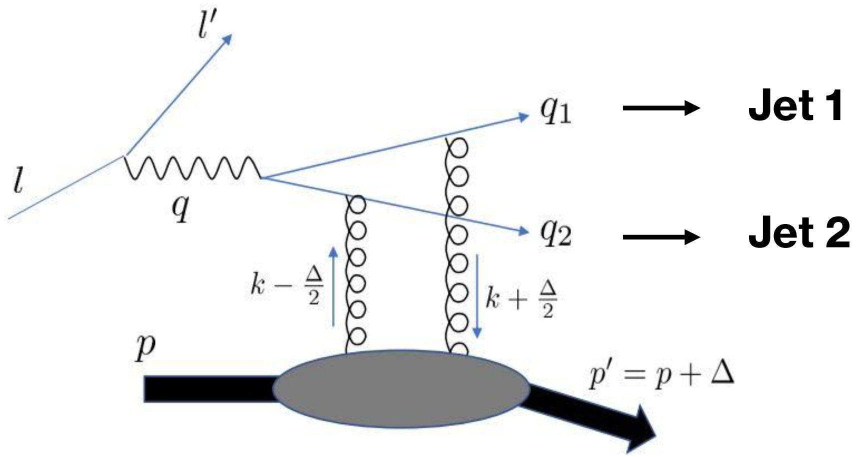
Big question:
Experimental observable?

Developments

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]



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Generalized TMDs and the exclusive double Drell-Yan process

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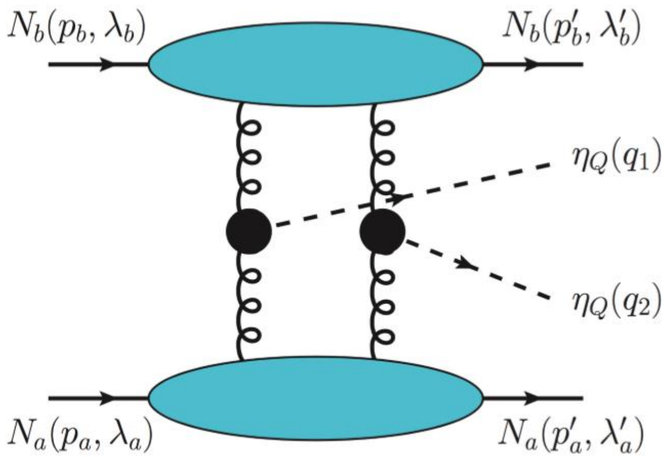
Exclusive double quarkonium production and generalized TM

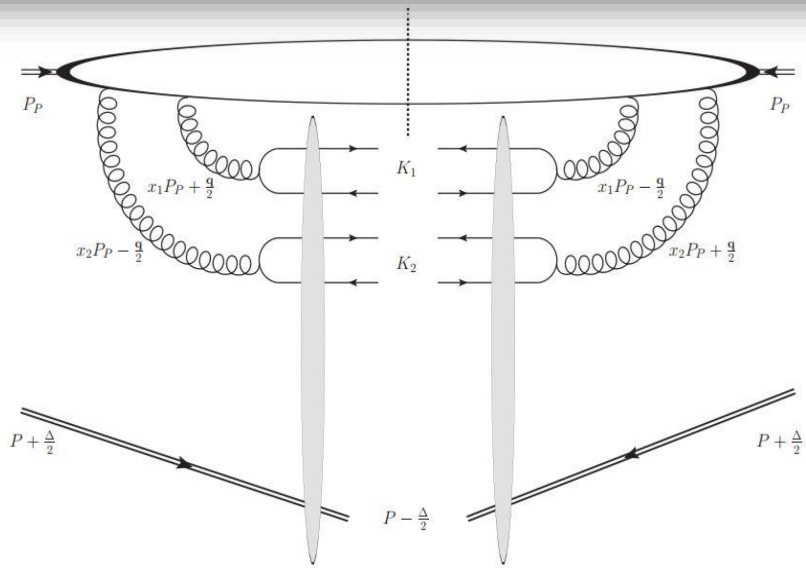
Shohini Bhattacharya,[1] Andreas Metz,[1] Vikash Kumar Ojha,[2] Jeng-Yuan Tsai,[1]

arXiv: 1807.08697 (2018)

Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions

Renaud Boussarie,[1] Yoshitaka Hatta,[2] Bo-Wen Xiao,[3,4] and Feng Yuan[5]





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Probing the Weizs ns

Renaud Bous

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target: GTMD distributions and the Odderons

Renaud Boussarie,[1] Yoshitaka Hatta,[1] Lech Szymanowski,[2] and Samuel Wallon[3,4]

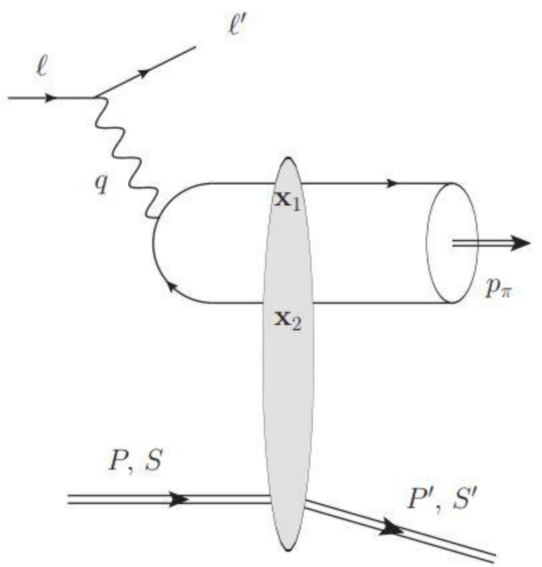

FIG. 1. Electroproduction of a π^0 meson at small x. The gray blob represents the interaction with the background field, and the white blob represents the Distribution Amplitude of the meson.

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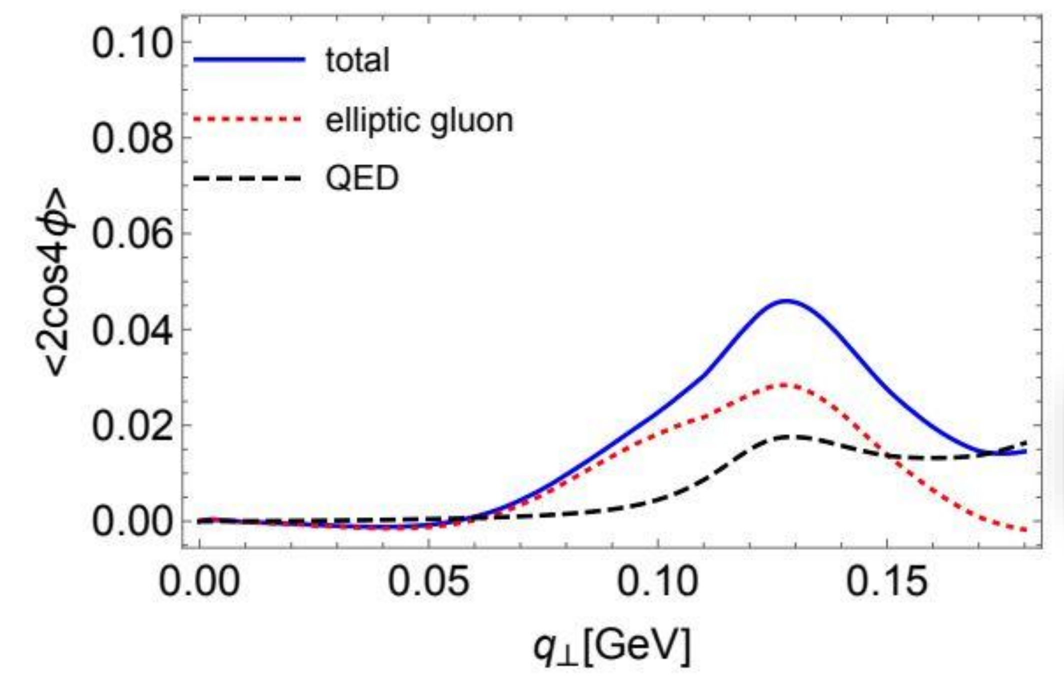

arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

Elliptic Wigner distribution contributes to $\cos 4\phi$ asymmetry

Developments

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Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

arXiv: 2201.08709 (2022/2024)

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]

Developments

arXiv: 1612.02438 (2016)

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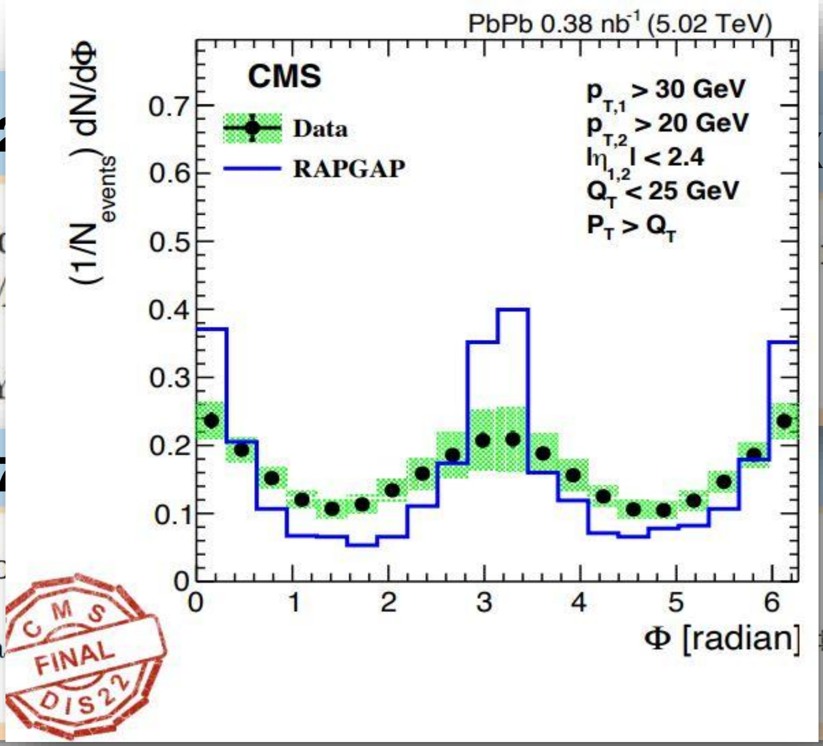
Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

arXiv: 2201.087

Signature o

Shohini Bhattacha



arXiv: 2205.00045 (2022)

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

Developments

arXiv: 1612.02438 (2016)

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Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]

arXiv: 1702.04387 (2017)

Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,[1] Andreas Metz,[1] and Jian Zhou[2]

arXiv: 1802.10550 (2018)

Exclusive double Drell-Yan:

Until now, this has been the sole known process sensitive to quark GTMDs

Renaud Boussarie,[1] Yoshitaka Hatta,[1] Lech Szymanowski,[2] and S

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

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Probing quark OAM through double Drell-Yan

Main findings

Physics Letters B 771 (2017) 396–400

Contents lists available at ScienceDirect

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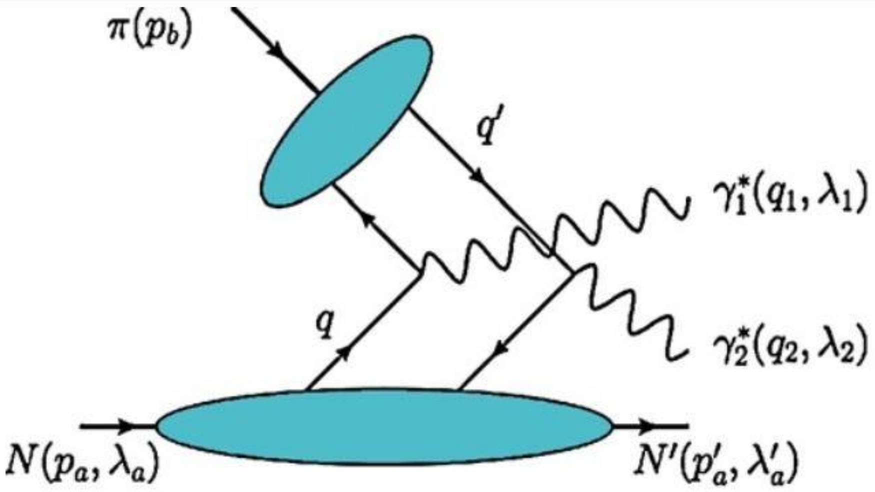
Generalized TMDs and the exclusive double Drell–Yan process

Shohini Bhattacharya [a], Andreas Metz [a,*], Jian Zhou [b]



Probing quark OAM through double Drell-Yan

Main findings

Example of an observable sensitive to quark OAM

$$\frac{1}{2}\left(\tau_{XY} - \tau_{YX}\right) \quad = \quad \frac{4}{M_a^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\,\mathrm{Re.}\left\{C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(+)}\left[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,\boldsymbol{F^*_{1,4}}\,\phi^*_\pi\right]\right.$$

Probing quark OAM through double Drell-Yan

Main findings

Example of an observable sensitive to quark OAM & spin-orbit correlation :

$$\frac{1}{2}\left(\tau_{XY} - \tau_{YX}\right) = \frac{4}{M_a^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\,\mathrm{Re.}\Big\{ C^{(-)}\Big[F_{1,1}\,\phi_\pi\Big] C^{(+)}\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, \boldsymbol{F^*_{1,4}}\, \phi_\pi^*\Big] - C^{(+)}\Big[G_{1,4}\,\phi_\pi\Big] C^{(-)}\Big[\vec{\beta}_\perp \cdot \vec{p}_{a\perp}\, \boldsymbol{G^*_{1,1}}\, \phi_\pi^*\Big]\Big\}$$

Probing quark OAM through double Drell-Yan

Main findings

Challenges:

- Low count rate (Amplitude $\sim \alpha_{em}^2$)

Probing quark OAM through double Drell-Yan

Main findings

Challenges:

- Low count rate (Amplitude $\sim \alpha_{em}^2$)
- Sensitivity to GTMDs only in the ERBL region $-\xi < x < \xi$

OAM density: $$L^{q/g}(x,\boldsymbol{\xi}) = -\int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x,k_\perp,\boldsymbol{\xi},\Delta_\perp = 0)$$

OAM: $$L^{q/g} = \int dx\, L^{q/g}(x,\boldsymbol{\xi = 0})$$

The challenge lies in extrapolating the distribution to the forward limit, where the OAM equation is applicable

Outline

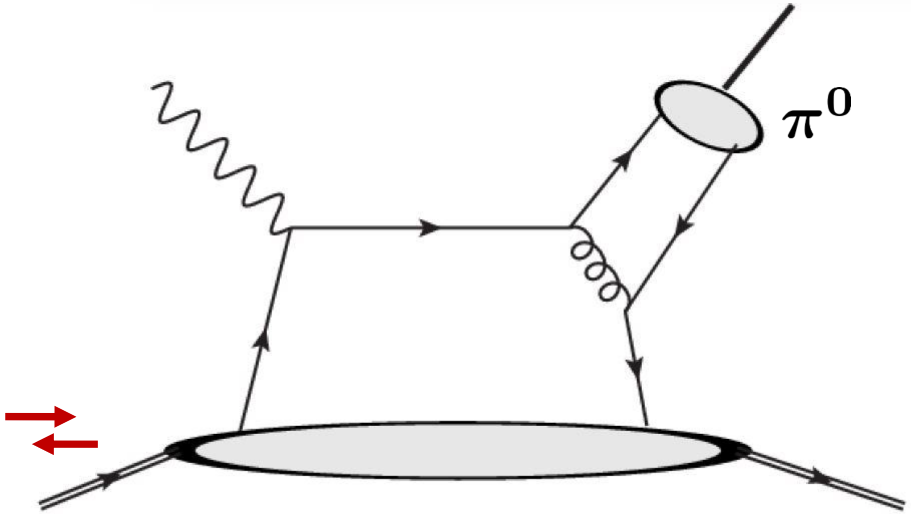
- Generalized TMDs & connection to spin physics
- Observable(s) for quark/gluon OAM & spin-orbit correlations:
    1) Exclusive pseudoscalar meson &
    2) Exclusive heavy (axial-) vector meson production
- Summary

Our work

PHYSICAL REVIEW LETTERS **133,** 051901 (2024)

Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive π^0 Production

Shohini Bhattacharya,[1] Duxin Zheng,[2] and Jian Zhou[3]

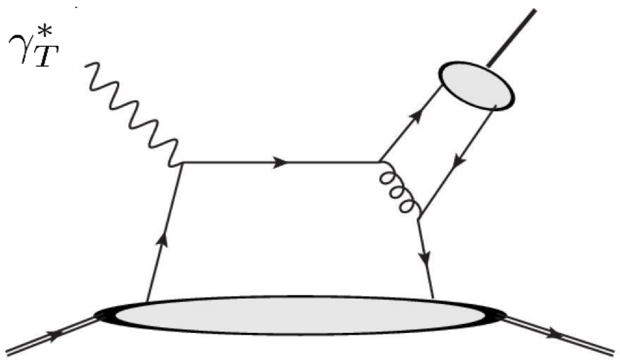


Main Observable:

Longitudinal single-target spin asymmetry

Probing quark OAM through π^0 production in ep collisions

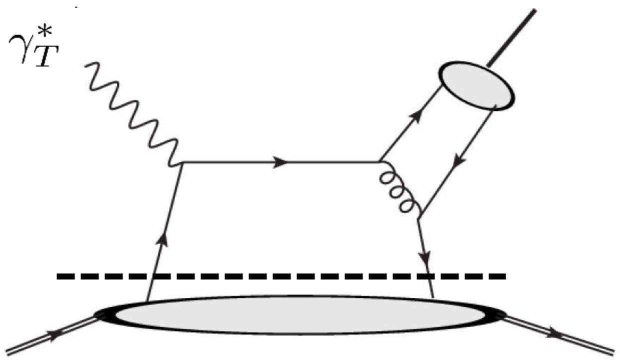
Scattering amplitude



4 leading-order Feynman diagrams

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2k_\perp\, H(x,\xi,z,k_\perp,\Delta_\perp)\, f^q(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

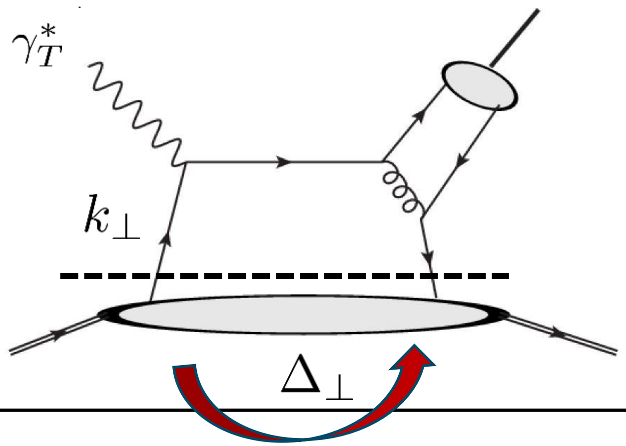
Hard part

Soft part from proton

Pion Distribution Amplitude

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

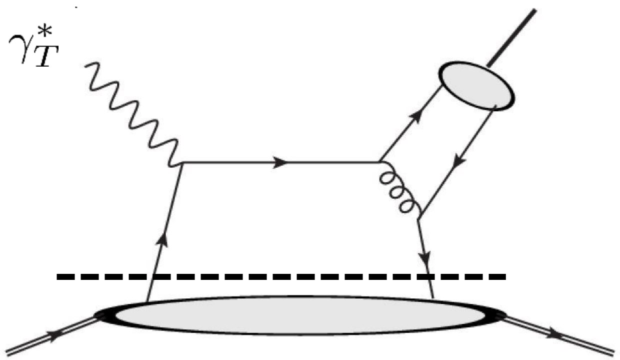
$$A \propto \int dx \int d^2k_\perp\, H(x,\xi,z,k_\perp,\Delta_\perp)\, f^q(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{\partial H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \frac{\partial H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \Delta_\perp^\mu + \ldots$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$\boxed{A \propto \int dx \int d^2k_\perp \, H(x, \xi, z, k_\perp, \Delta_\perp) \, f^q(x, \xi, k_\perp, \Delta_\perp) \int dz \phi_\pi(z)}$$

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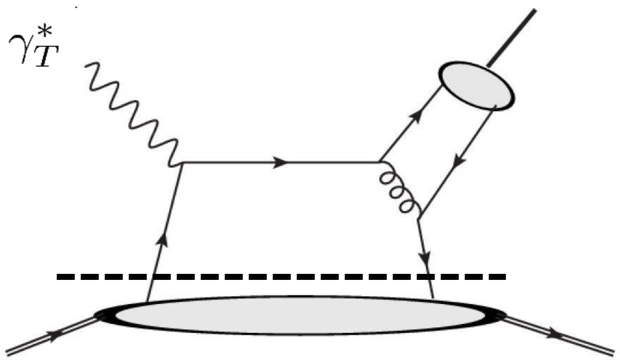
Twist 2 term vanishes

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

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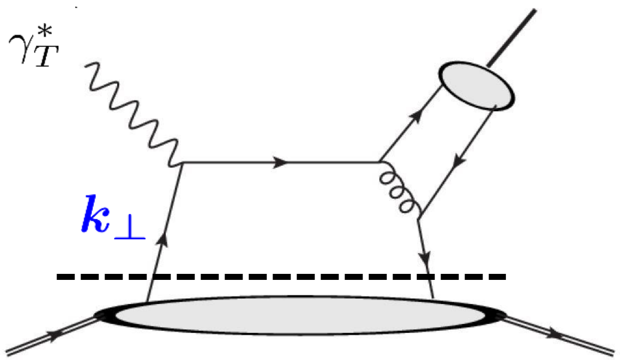
Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

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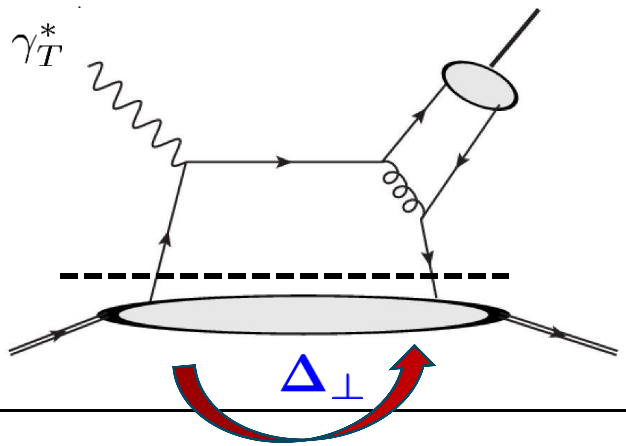
$$\boxed{A \propto \int d^2k_\perp k_\perp^2 \text{ GTMD}}$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int \boldsymbol{d^2 k_\perp}\, H(x,\xi,z,k_\perp,\Delta_\perp)\, \boldsymbol{f^q}(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

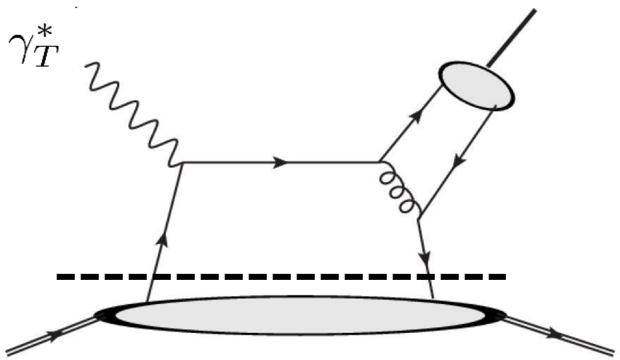
Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{\partial H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \frac{\partial H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \boldsymbol{\Delta_\perp^\mu} + \ldots$$

$$\boxed{A \propto \mathbf{GPD}}$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2k_\perp\, H(x,\xi,z,k_\perp,\Delta_\perp)\, f^q(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

Collinear twist-expansion of hard part:

The scattering amplitudes are a convolution of **moments of GTMDs** and **GPDs** and are of twist-3 nature

This interplay is itself physically significant, as it connects two complementary descriptions of hadron structure and therefore offers a valuable opportunity to simultaneously constrain both GTMD and GPD dynamics within a unified exclusive framework.

Probing quark OAM through π^0 production in ep collisions

Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \times \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda,-\lambda'} \frac{M\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{S}_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\} \qquad S_\perp^\mu = (0^+, 0^-, -i, \lambda)$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}$$

Probing quark OAM through π^0 produc…

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$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \boxed{\{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}}$$

Compton Form Factors (CFFs):

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2 \int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \tag{8}$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \tag{9}$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx x \frac{\xi(1-\xi^2)\int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \tag{10}$$

$$\mathcal{G}_{1,2} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)(1-\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \tag{11}$$

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi\int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \tag{12}$$

$$\mathcal{G}_{1,4} = \int_{-1}^{1} dx \int_0^1 dz \frac{x(4\xi^2 z+\xi^2-2x^2z+x^2)}{z^2\xi(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\phi_\pi(z) \times \int d^2k_\perp G_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp). \tag{13}$$

Probing quark OAM through π^0 production in ep collisions

Angular correlations

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$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \boxed{\{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}}$$

Sensitivity to quark OAM

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2 \int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx\, x \frac{\xi(1-\xi^2)\int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (10)$$

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Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\times \left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right] + \cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.$$

$$\left. +\lambda \sin(2\phi)\, 2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Unpolarized

Single spin asymmetry

$$a = \frac{2(1-y)}{1+(1-y)^2}$$

Probing quark OAM through π^0 production in ep collisions

Cross section

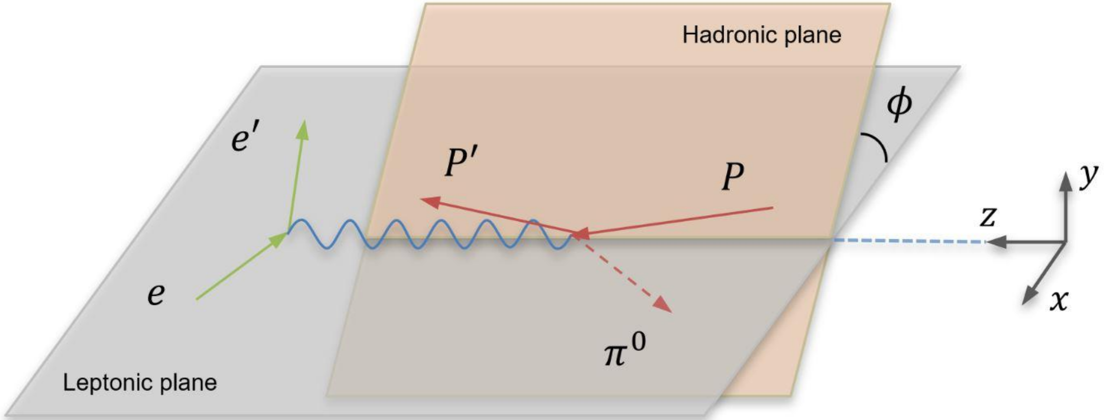
$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.$$

$$\boxed{\left.+\lambda\sin(2\phi)\, 2a\,\mathrm{Re}\left[\left(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4}\right)\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}}$$

Distinguished experimental signature of quark OAM

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$



Probing quark OAM through π^0 production in ep collisions

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$$\boxed{\times \left\{ \left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 \right] \right.}$$

$$\left. + \lambda \sin(2\phi)\, 2a\, \mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4}) \left(\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^* \right) \right] \right\}$$

Surprise!

- Probe quark Sivers function through an unpolarized target

$$\boxed{\mathrm{Im}\left[F_{1,2} \right]\Big|_{\Delta=0} = -f_{1T}^{\perp}}$$

(Similar to the gluon GTMD $F_{1,2}$, as discussed in Boussarie, Hatta, Szymanowski, Wallon, 2019)

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\boxed{\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\boldsymbol{\mathcal{F}_{1,2}}+\boldsymbol{\mathcal{G}_{1,2}}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.}$$

$$\left.+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Surprise!

- Probe quark Sivers function through an unpolarized target $\boxed{\mathrm{Im}\left[\boldsymbol{F_{1,2}}\right]\big|_{\Delta=0} = -\boldsymbol{f_{1T}^{\perp}}}$
- Probe quark worm gear function through an unpolarized target $\boxed{\mathrm{Re}\left[\boldsymbol{G_{1,2}}\right]\big|_{\Delta=0} = \boldsymbol{g_{1T}}}$

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2\xi^3\boldsymbol{\Delta_\perp^2}}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\boxed{\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\boldsymbol{\Delta_\perp^2}}|\boldsymbol{\mathcal{F}_{1,2}}+\boldsymbol{\mathcal{G}_{1,2}}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.}$$

$$\left.+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Helicity flip terms persist even when $\Delta_\perp \to 0$

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi}=\frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2f_\pi^2\xi^3\Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$
$$\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.$$
$$\left.+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Since both unpolarized and polarized cross sections contribute at twist-3, the magnitudes of the asymmetries are not power-suppressed

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

End-point singularity

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

Model-dependent method:

$$\boxed{\int_{\langle p_\perp^2\rangle/Q^2}^{1-\langle p_\perp^2\rangle/Q^2} dz}$$

$\langle p_\perp^2 \rangle = 0.04\,\mathrm{GeV}^2$ determined based on a fit to CLAS data

S. V. Goloskokov and P. Kroll, 2005

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2 \boldsymbol{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

Model-dependent method:

$$\int_{\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2}^{1-\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2} dz$$

S. V. Goloskokov and P. Kroll, 2005

$$\boxed{\frac{1}{(x-\xi+i\epsilon)^2} \rightarrow \frac{1}{(x-\xi-\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2+i\epsilon)^2}}$$

I. V. Anikin, O. V. Teryaev, 2003

Probing quark OAM through π^0 production in ep collisions

Numerical results

Kinematics:

	$Q^2(\mathrm{GeV}^2)$	$\sqrt{s}_{ep}(\mathrm{GeV})$
EIC	10	100
EicC	3	16

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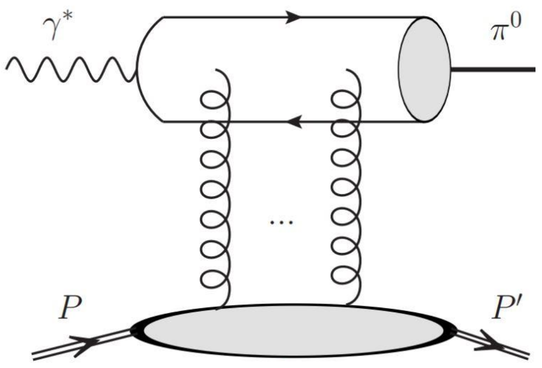
- We focus on large skewness (ξ) region to suppress gluon contribution



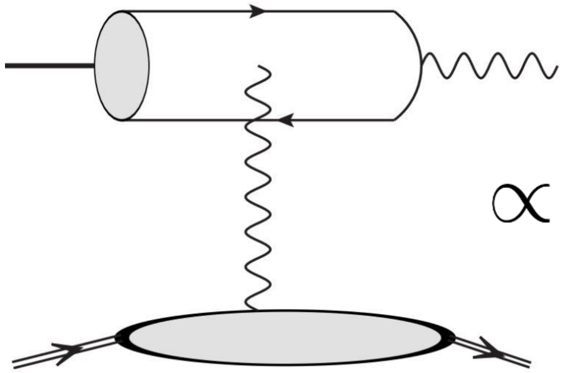
Probing quark OAM through π^0 production in ep collisions

Numerical results

Kinematics:

	$Q^2(\mathrm{GeV}^2)$	$\sqrt{s_{ep}}(\mathrm{GeV})$
EIC	10	100
EicC	3	16

- We focus on large skewness (ξ) region to suppress gluon contribution
- We focus on large momentum transfer (t) region to suppress contribution from Primakoff process



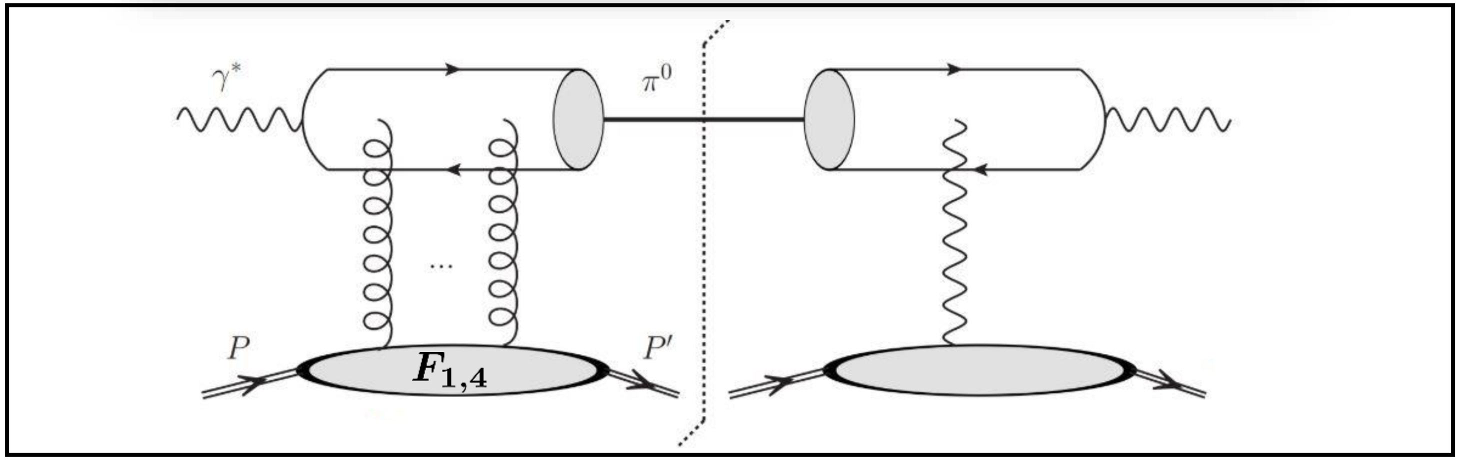
$$\propto \frac{1}{t}$$

Probing quark OAM through π^0 production in ep collisions

Remark:

Accessing the gluon GTMD $F_{1,4}$ in exclusive π^0 production in ep collisions

Shohini Bhattacharya,[1] Duxin Zheng,[2] and Jian Zhou[3]



$$\frac{d\Delta\sigma}{dt\,dQ^2\,dx_B\,d\phi} = -\sin(2\phi)\frac{\alpha_{em}^3\alpha_s f_\pi^2(1-y)\xi x_B\mathcal{F}(t)}{3Q^8N_c}\left[\int_0^1 dz\,\frac{\phi_\pi(z)}{z(1-z)}\right]^2 \mathrm{Im}\left[\int_{-1}^1 dx\,\frac{F_{1,4}^{(1)}(x,\xi,\Delta_\perp)/M^2}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\right]$$

The same azimuthal asymmetry, precisely mirroring what we observe in this study, also emerges from the interference between the Primakoff process and the contribution from the gluon GTMD

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\ \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$\boxed{H^q(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(\beta+\xi\alpha-x) \times \frac{3}{4}|\beta|^{-1.3\,t}\frac{[(1-|\beta|)^2-\alpha^2]}{(1-|\beta|)^3}q(|\beta|)}$$

JAM 2022 PDFs

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

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The t-dependence is determined based on a fit to CLAS data

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\ \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

- Model for OAM:

    1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L^q_{can}(\boldsymbol{x}) = x\int_x^1 \frac{dx'}{x'}\, q(x') - x\int_x^1 \frac{dx'}{x'^2}\, \Delta q(x') + \text{ genuine twist-three}$$

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

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Probing quark OAM through π^0 production in ep collisions

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  2. Use the Double distribution approach to construct $xL^q(x,\boldsymbol{\xi})e^{\boldsymbol{t}/\Lambda}$ from $xL^q(x)$

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

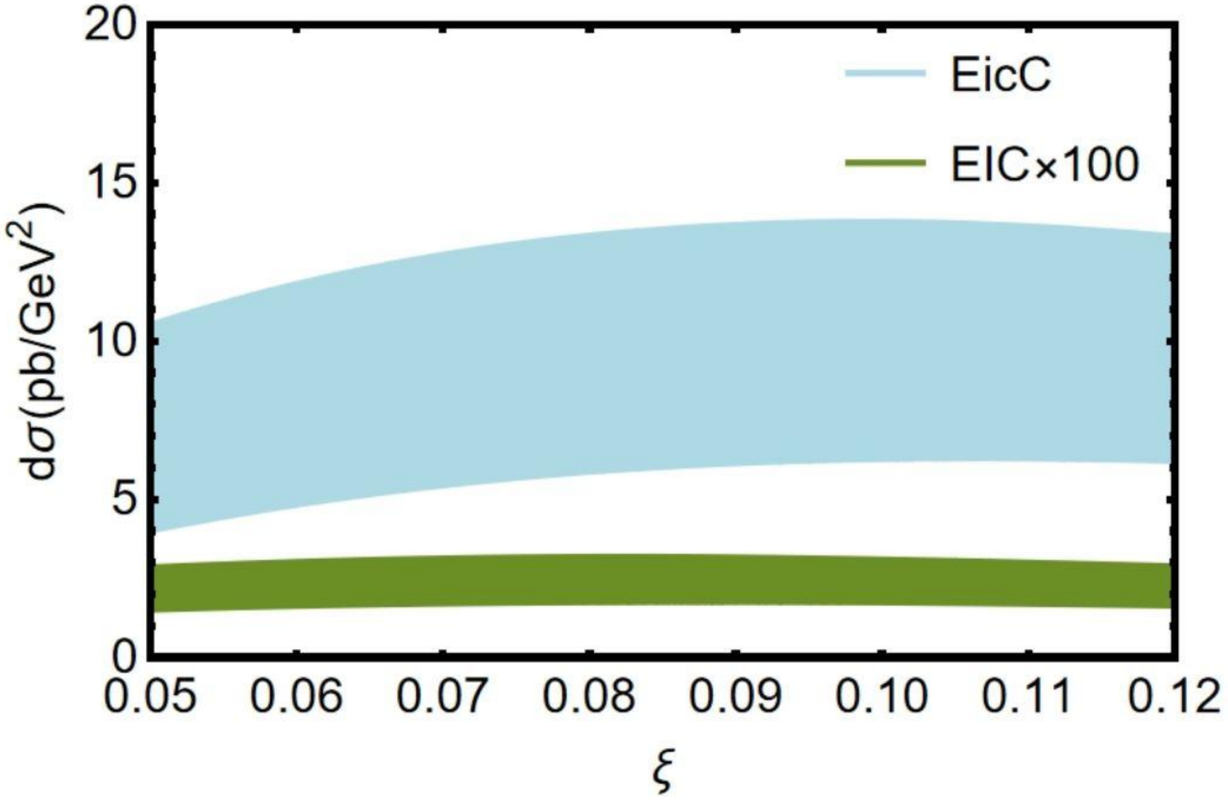
- Pion distribution amplitude:

Asymptotic form $\boxed{\phi_\pi(z) = 6z(1-z)}$

Probing quark OAM through π^0 production in ep collisions

Numerical results

Unpolarized cross section



Findings:

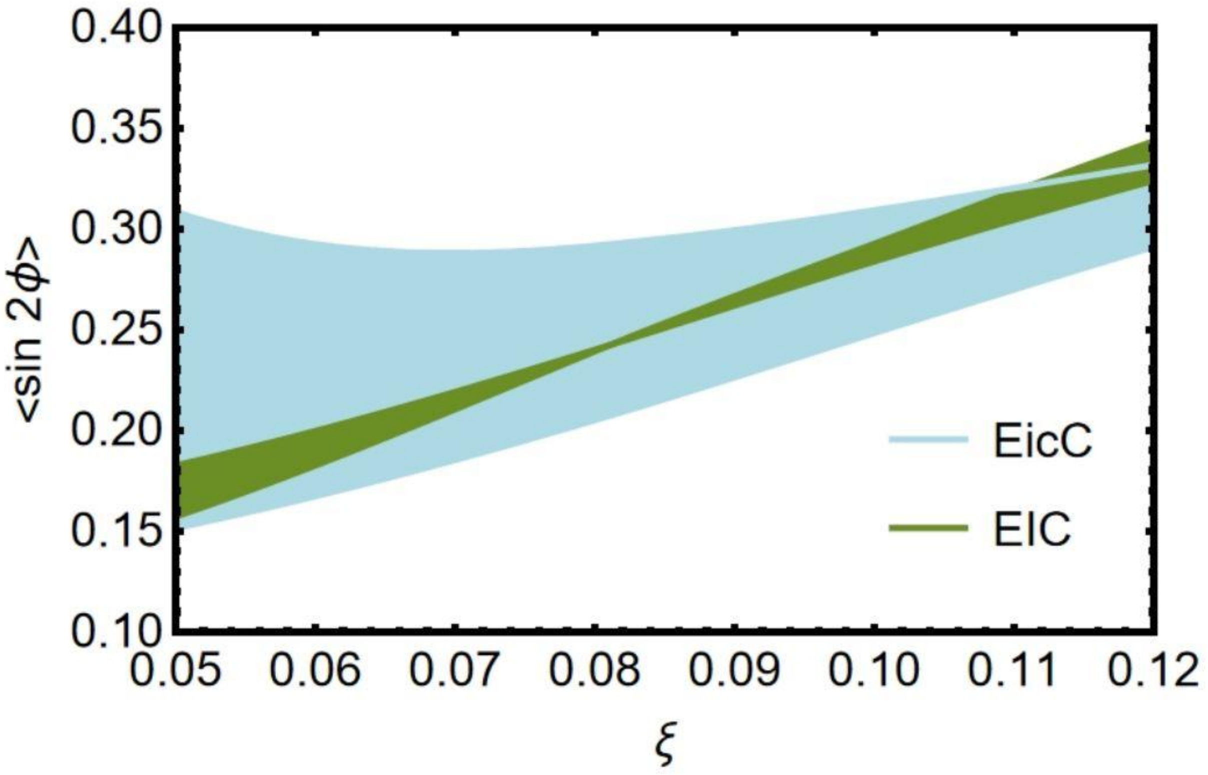
- The unpolarized cross section exhibits a notable magnitude at EicC energy
- Relatively small at EIC energy

This strong suppression with increasing Q^2 suggests that it may be particularly interesting to study this reaction in the kinematic range accessible at JLab.

Probing quark OAM through π^0 production in ep collisions

Numerical results

Asymmetry



$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P.S.}} \sin(2\phi)\, d\mathcal{P.S.}}{\int \frac{d\sigma}{d\mathcal{P.S.}} d\mathcal{P.S.}}$$

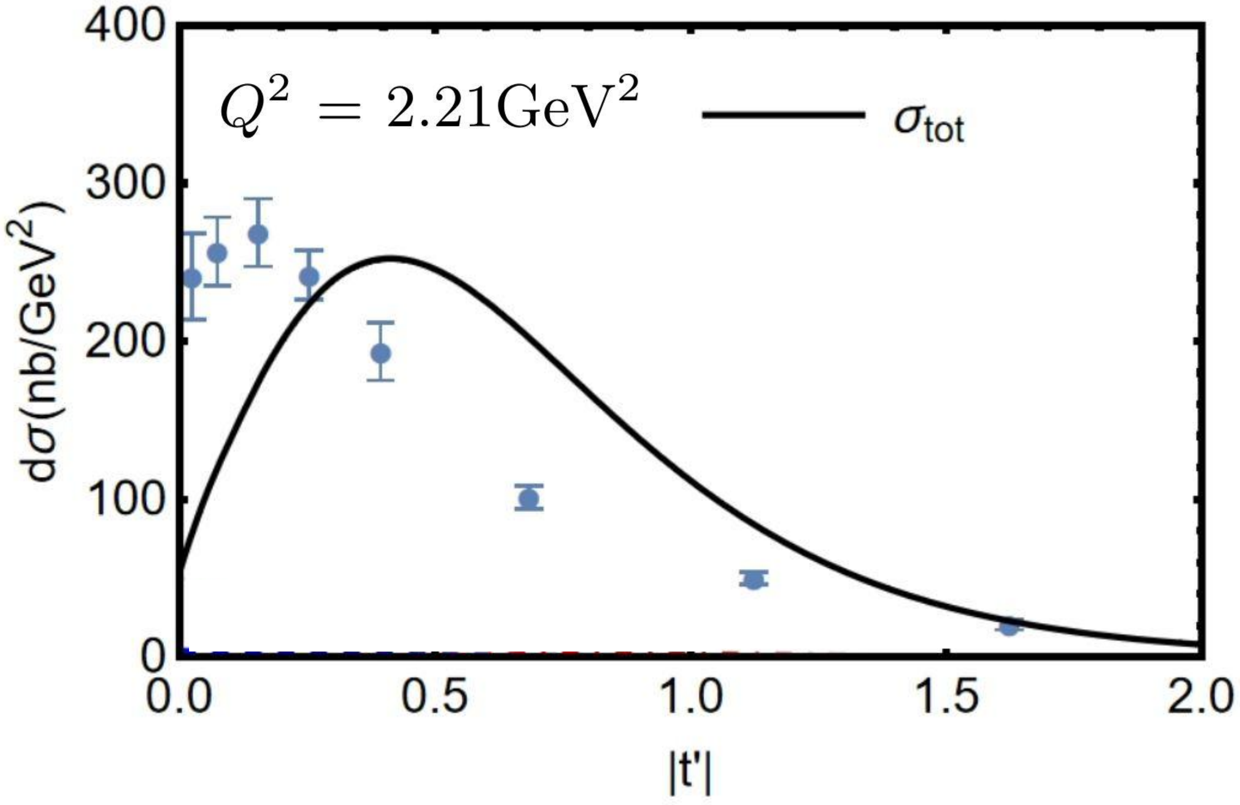
Findings:

The asymmetries are substantial for both EIC & EicC kinematics

Probing quark OAM through π^0 production in ep collisions

Numerical results

Comparison with CLAS data (I. Bedlinskiy et al., Phys. Rev. C 90, 025205 (2014))



Unpolarized cross section:

$$\boxed{\frac{d\sigma_T}{dt} + a\frac{d\sigma_L}{dt}}$$

Probing quark OAM through π^0 production in ep collisions

Numerical results

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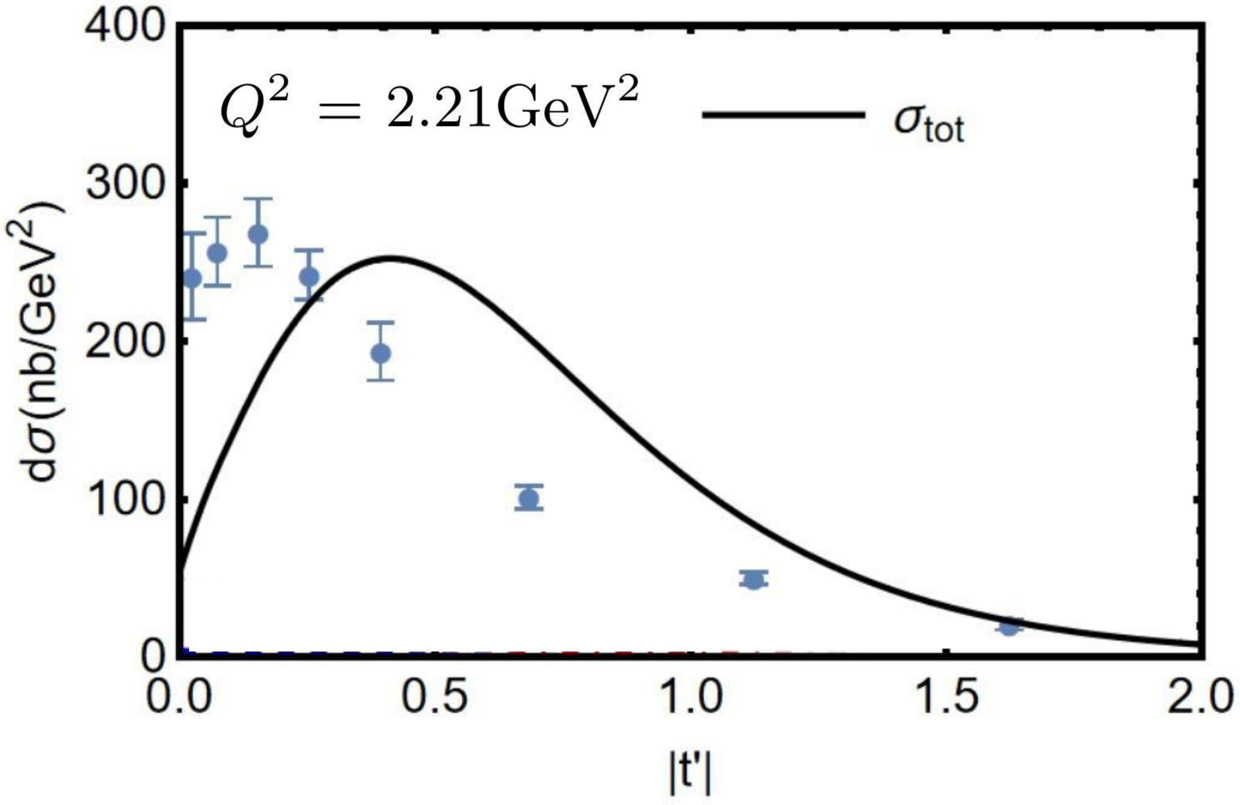


Findings:

- Our theoretical model is in reasonable agreement with experimental data

More opportunities with CLAS's recent data on single spin asymmetry

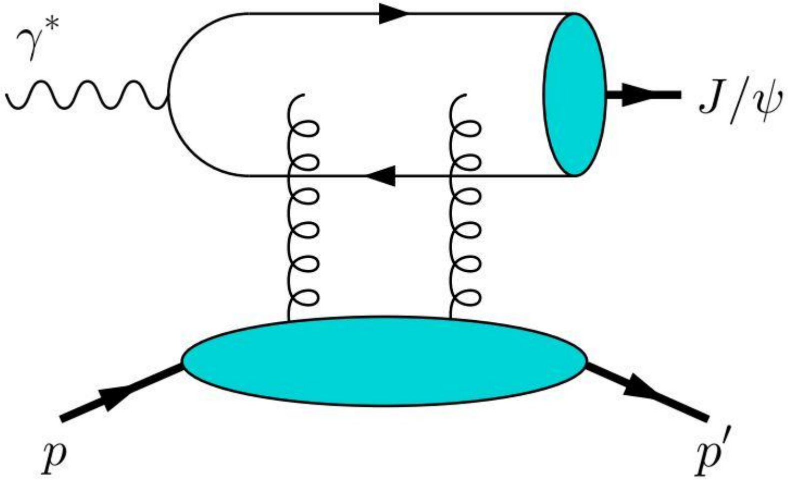
Outline

- Generalized TMDs & connection to spin physics
- Observable(s) for quark/gluon OAM & spin-orbit correlations:
  1) Exclusive pseudoscalar meson &
  2) Exclusive heavy (axial-) vector meson production
- Summary

Our work

Gluon Generalized TMD signatures at the EIC from exclusive heavy (axial-)vector meson production

Shohini Bhattacharya,[1,*] David DeAngelo,[1,†] Lei Yang,[2,‡] Duxin Zheng,[3,§] and Jian Zhou[2,4,¶]



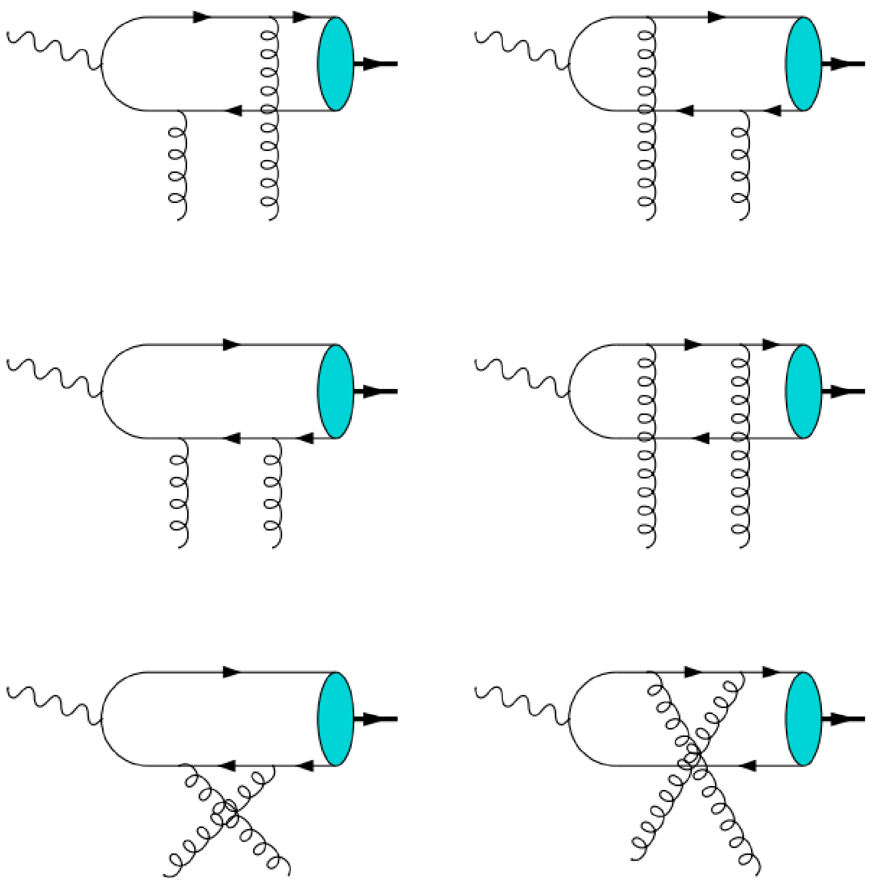
Main Observable:

Interference between different virtual-photon polarizations leads to distinct azimuthal modulations, including cos 2ϕ and sin 2ϕ terms, that are sensitive to gluon OAM and spin-orbit correlations

Probing gluon OAM & spin-orbit correlations via vector-mesons

Probing gluon OAM & spin-orbit correlations via vector-mesons

Scattering amplitude



6 leading-order Feynman diagrams

Probing gluon OAM & spin-orbit correlations via vector-mesons

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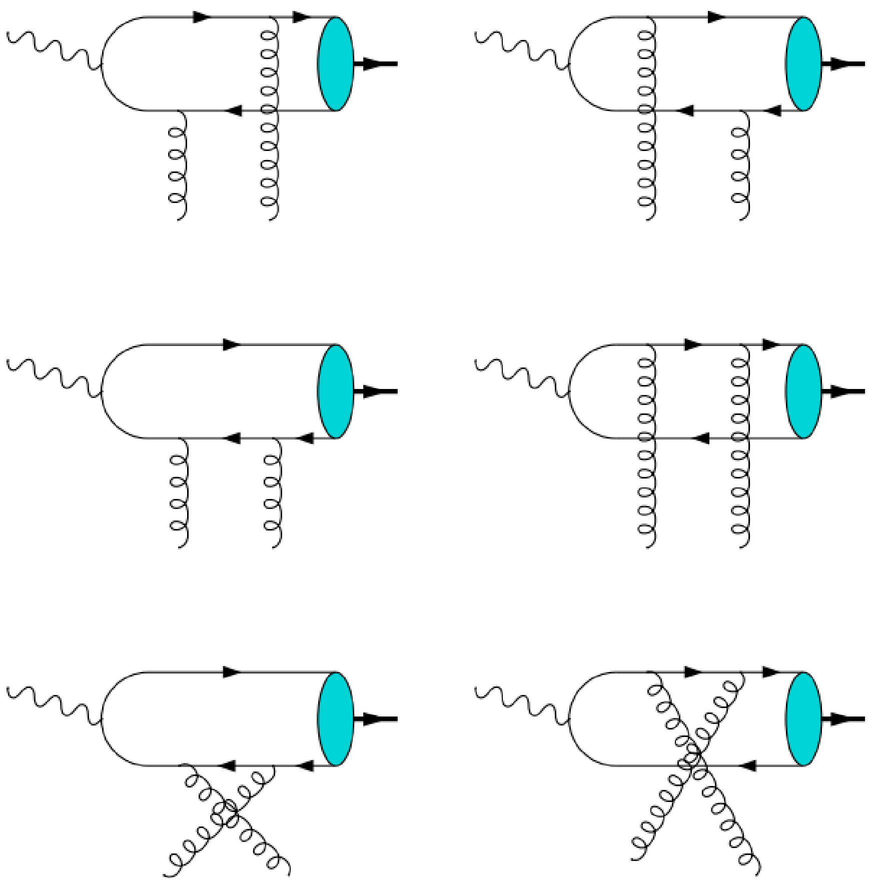


6 leading-order Feynman diagrams

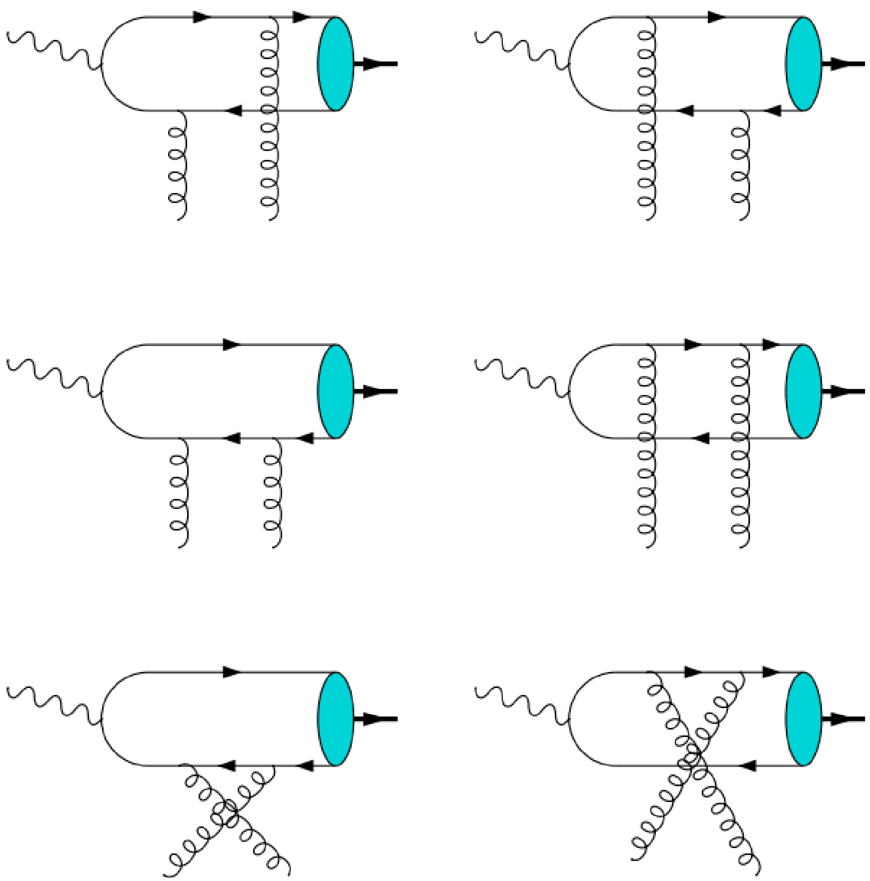
Scattering amplitude:

$$\boxed{\mathcal{M} \sim \int dz \phi_V(z) \int dx \frac{1}{x-\xi+i\epsilon} \frac{1}{x+\xi-i\epsilon} \int d^2k_\perp H^{ij} W^{g[ij]}}$$

- Upon performing the collinear twist expansion of the hard scattering kernel, we find contributions from both twist-2 and twist-3 terms.

Probing gluon OAM & spin-orbit correlations via vector-mesons

Scattering amplitude



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- Upon performing the collinear twist expansion of the hard scattering kernel, we find contributions from both twist-2 and twist-3 terms.
- Calculated scattering amplitudes for specific polarization configurations of the virtual photon and the vector meson:

Notation: $\mathcal{M}^{\gamma^*, M}$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^*= Photon Polarization
M=Meson Polarization

Proton helicity-conserving amplitudes (z=1/2):

$$\begin{aligned}
\mathcal{M}^{LL} &= C\delta_{\lambda,\lambda'}\left\{\mathcal{H}^{LL}_{eff}\right\} \\
\mathcal{M}^{TT} &= C\delta_{\lambda,\lambda'}\left(\epsilon^{\gamma^*}_{\perp}\cdot\epsilon^{V}_{\perp}\right)\left\{\mathcal{H}^{TT}_{eff}\right\} \\
\mathcal{M}^{TL} &= C\delta_{\lambda,\lambda'}\left\{\left(\epsilon^{\gamma^*}_{\perp}\cdot\Delta_\perp\right)\left(\mathcal{H}^{TL}_{eff}+\mathcal{F}^{TL}_{1,1}\right)+\lambda\left(\epsilon^{\gamma^*}_{\perp}\times\Delta_\perp\right)\mathcal{F}^{TL}_{1,4}\right. \\
&\qquad \left.+\left(\epsilon^{\gamma^*}_{\perp}\cdot\Delta_\perp\right)\mathcal{G}^{TL}_{1,1}-\lambda\left(\epsilon^{\gamma^*}_{\perp}\times\Delta_\perp\right)\left(\mathcal{H}'^{TL}_{eff}+\mathcal{G}^{TL}_{1,4}\right)\right\} \\
\mathcal{M}^{LT} &= 0
\end{aligned}$$

$$C=\frac{16\,i\,\pi\,e\,e_q\,\alpha_s f_V\,m_V}{N_c\,(m_V^2+Q^2)}$$

CFFs:

$$\mathcal{H}^{LL}_{eff}=-\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\frac{Q\sqrt{1-\xi^2}}{m_V}H^g_{eff}$$

$$\mathcal{H}^{TT}_{eff}=\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\sqrt{1-\xi^2}H^g_{eff}$$

$$\mathcal{H}^{TL}_{eff}=\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{\xi^2\sqrt{1-\xi^2}}{m_V}H^g_{eff}$$

$$\mathcal{H}'^{TL}_{eff}=\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{ix\xi\sqrt{1-\xi^2}}{m_V}H'^g_{eff}$$

$$\mathcal{F}^{TL}_{1,1}=\int_{-1}^{1}\frac{dx}{(x-\xi-i\varepsilon)^2(x+\xi+i\varepsilon)^2}\frac{2x\xi}{m_V}\frac{1}{\sqrt{1-\xi^2}}\int d^2k_\perp\left(\frac{k_\perp\cdot\Delta_\perp}{\Delta_\perp^2}\right)F^g_{1,1}$$

$$\mathcal{G}^{TL}_{1,1}=-\frac{1}{\sqrt{1-\xi^2}}\int_{-1}^{1}dx\frac{\xi^2}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_\perp\left(\frac{k_\perp^2}{M^2}\right)G^g_{1,1}$$

$$\mathcal{F}^{TL}_{1,4}=\frac{1}{\sqrt{1-\xi^2}}\int_{-1}^{1}dx\frac{ix\xi}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_\perp\left(\frac{k_\perp^2}{M^2}\right)F^g_{1,4}$$

$$\mathcal{G}^{TL}_{1,4}=\int\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{2i\xi^2}{m_V}\frac{1}{\sqrt{1-\xi^2}}\int d^2k_\perp\left(\frac{k_\perp\cdot\Delta_\perp}{\Delta_\perp^2}\right)G^g_{1,4}$$

$$H^g_{eff}=H^g-\frac{\xi^2}{1-\xi^2}E^g$$

$$H'^g_{eff}=\widetilde{H}^g-\frac{\xi^2}{1-\xi^2}\widetilde{E}^g$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^*= Photon Polarization
M=Meson Polarization

Gluon OAM

Proton helicity-conserving amplitudes (z=1/2):

$$
\begin{aligned}
\mathcal{M}^{LL} &= C\delta_{\lambda,\lambda'}\left\{\mathcal{H}^{LL}_{eff}\right\} \\
\mathcal{M}^{TT} &= C\delta_{\lambda,\lambda'}\left(\epsilon^{\gamma^*}_{\perp}\cdot\epsilon^{V}_{\perp}\right)\left\{\mathcal{H}^{TT}_{eff}\right\} \\
\mathcal{M}^{TL} &= C\delta_{\lambda,\lambda'}\left\{\left(\epsilon^{\gamma^*}_{\perp}\cdot\Delta_{\perp}\right)\left(\mathcal{H}^{TL}_{eff}+\mathcal{F}^{TL}_{1,1}\right)+\lambda\left(\epsilon^{\gamma^*}_{\perp}\times\Delta_{\perp}\right)\mathcal{F}^{TL}_{1,4}\right. \\
&\quad \left.+\left(\epsilon^{\gamma^*}_{\perp}\cdot\Delta_{\perp}\right)\mathcal{G}^{TL}_{1,1}-\lambda\left(\epsilon^{\gamma^*}_{\perp}\times\Delta_{\perp}\right)\left(\mathcal{H}'^{TL}_{eff}+\mathcal{G}^{TL}_{1,4}\right)\right\} \\
\mathcal{M}^{LT} &= 0
\end{aligned}
$$

$$C = \frac{16\, i\, \pi\, e\, e_q\, \alpha_s\, f_V\, m_V}{N_c\,(m_V^2+Q^2)}$$

Gluon's spin-orbit correlations

CFFs:

$$\mathcal{H}^{LL}_{eff} = -\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\frac{Q\sqrt{1-\xi^2}}{m_V}H^g_{eff}$$

$$\mathcal{H}^{TT}_{eff} = \int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\sqrt{1-\xi^2}H^g_{eff}$$

$$\mathcal{H}^{TL}_{eff} = \int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{\xi^2\sqrt{1-\xi^2}}{m_V}H^g_{eff}$$

$$\mathcal{H}'^{TL}_{eff} = \int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{ix\xi\sqrt{1-\xi^2}}{m_V}H'^g_{eff}$$

$$\mathcal{F}^{TL}_{1,1} = \int_{-1}^{1}\frac{dx}{(x-\xi-i\varepsilon)^2(x+\xi+i\varepsilon)^2}\frac{2x\xi}{m_V}\frac{1}{\sqrt{1-\xi^2}}\int d^2k_{\perp}\left(\frac{k_{\perp}\cdot\Delta_{\perp}}{\Delta^2_{\perp}}\right)F^g_{1,1}$$

$$\mathcal{G}^{TL}_{1,1} = -\frac{1}{\sqrt{1-\xi^2}}\int_{-1}^{1}dx\frac{\xi^2}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_{\perp}\left(\frac{k^2_{\perp}}{M^2}\right)G^g_{1,1}$$

$$\mathcal{F}^{TL}_{1,4} = \frac{1}{\sqrt{1-\xi^2}}\int_{-1}^{1}dx\frac{ix\xi}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_{\perp}\left(\frac{k^2_{\perp}}{M^2}\right)F^g_{1,4}$$

$$\mathcal{G}^{TL}_{1,4} = \int\frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2}\frac{2i\xi^2}{m_V}\frac{1}{\sqrt{1-\xi^2}}\int d^2k_{\perp}\left(\frac{k_{\perp}\cdot\Delta_{\perp}}{\Delta^2_{\perp}}\right)G^g_{1,4}$$

$$H^g_{eff} = H^g - \frac{\xi^2}{1-\xi^2}E^g$$

$$H'^g_{eff} = \widetilde{H}^g - \frac{\xi^2}{1-\xi^2}\widetilde{E}^g$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^*= Photon Polarization
M=Meson Polarization

Proton helicity-non conserving amplitudes (z=1/2):

$$\begin{aligned}
\widetilde{\mathcal{M}}^{LL} &= C\delta_{\lambda,-\lambda'}\left\{(\Delta_\perp \times S_\perp)\,\widetilde{\mathcal{E}}^{LL}\right\}\\
\widetilde{\mathcal{M}}^{TT} &= C\delta_{\lambda,-\lambda'}(\epsilon_\perp^{V}\cdot\epsilon_\perp^{\gamma*})\left\{(\Delta_\perp \times S_\perp)\,\widetilde{\mathcal{E}}^{TT}\right\}\\
\widetilde{\mathcal{M}}^{TL} &= C\delta_{\lambda,-\lambda'}\left\{(\epsilon_\perp^{\gamma*}\times S_\perp)\,\widetilde{\mathcal{F}}^{TL}_{1,2} - (\epsilon_\perp^{\gamma*}\times S_\perp)\,\widetilde{\mathcal{G}}^{TL}_{1,2}\right\}\\
\widetilde{\mathcal{M}}^{LT} &= 0
\end{aligned}$$

$$C = \frac{16\, i\, \pi\, e\, e_q\, \alpha_s f_V\, m_V}{N_c\,(m_V^2 + Q^2)}$$

CFFs:

$$\widetilde{\mathcal{E}}^{LL} = -\int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\frac{Q}{m_V}\frac{1}{2M\sqrt{1-\xi^2}}E^g$$

$$\widetilde{\mathcal{E}}^{TT} = \int_{-1}^{1}\frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\frac{1}{2M\sqrt{1-\xi^2}}E^g$$

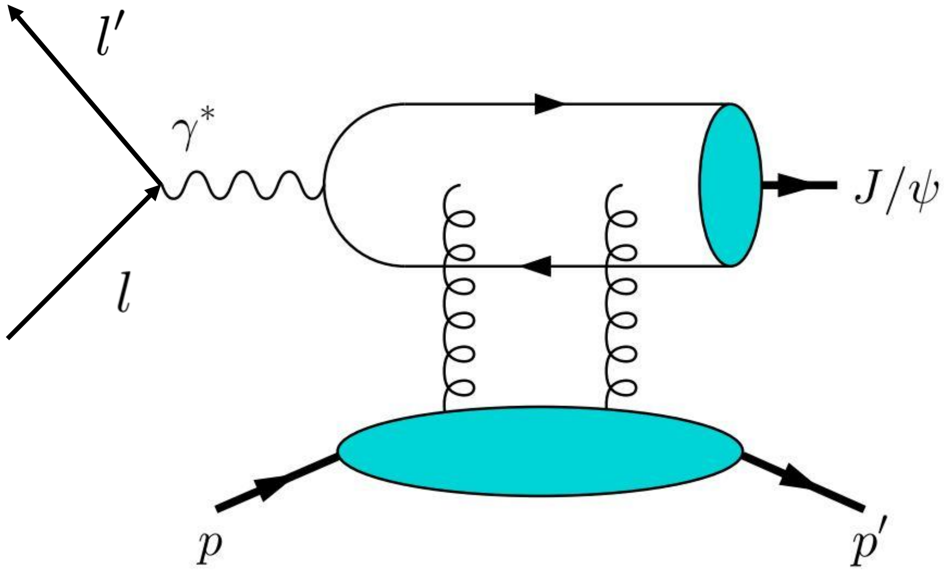
$$\widetilde{\mathcal{F}}^{TL}_{1,2} = M\sqrt{1-\xi^2}\int_{-1}^{1}dx\frac{x\xi}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_\perp\left(\frac{k_\perp^2}{M^2}\right)F^g_{1,2}$$

$$\widetilde{\mathcal{G}}^{TL}_{1,2} = -M\sqrt{1-\xi^2}\int_{-1}^{1}dx\frac{\xi^2}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\int d^2k_\perp\left(\frac{k_\perp^2}{M^2}\right)G^g_{1,2}$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

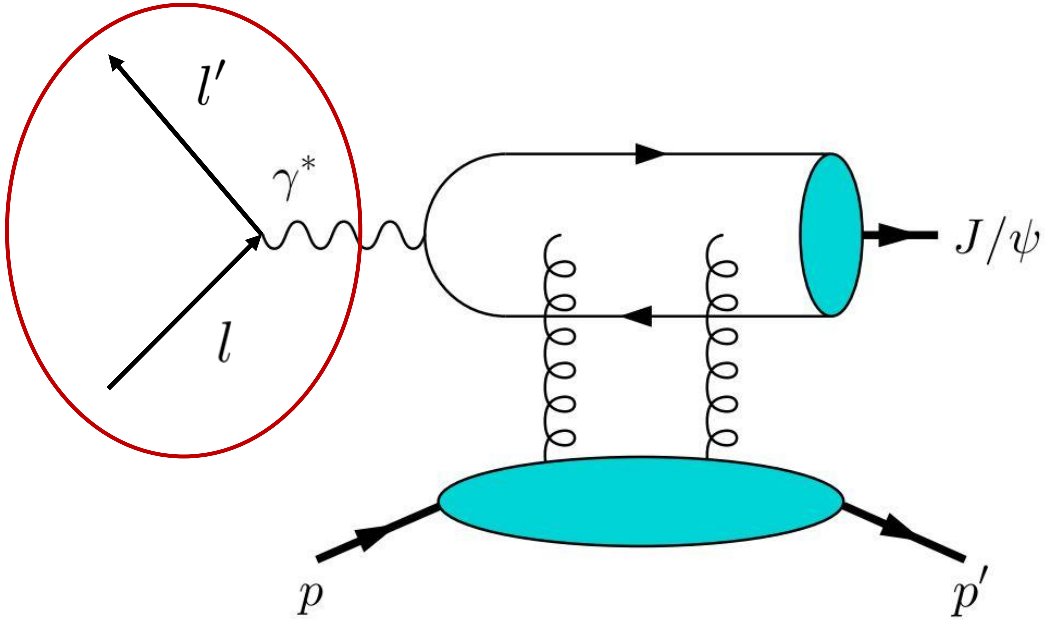
$$d\sigma = \frac{\mathcal{C}}{2s} L_{\mu\nu} \mathcal{M}^{\mu} \mathcal{M}^{*\nu}$$



Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$d\sigma = \frac{\mathcal{C}}{2s} L_{\mu\nu} \mathcal{M}^{\mu} \mathcal{M}^{*\nu}$$



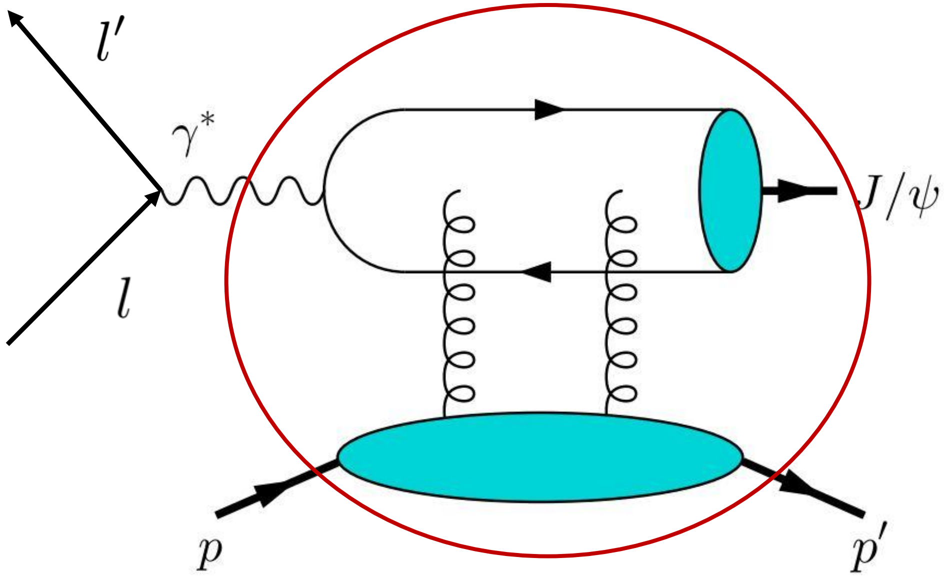
Unpolarized leptonic tensor:

$$L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu) - g_{\mu\nu} Q^2$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$d\sigma = \frac{\mathcal{C}}{2s} L_{\mu\nu} \mathcal{M}^{\mu} \mathcal{M}^{*\nu}$$



Unpolarized leptonic tensor:

$$L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu) - g_{\mu\nu} Q^2$$

Hadronic tensor (photon-helicity amplitudes):

$$\mathcal{M}^{\mu} = \sum_{\lambda=0,\pm} \epsilon^{\mu}_{\lambda} \mathcal{M}^{\gamma^* p}_{\lambda}$$

$$\mathcal{M}^{*\nu} = \sum_{\lambda=0,\pm} \epsilon^{*\nu}_{\lambda} (\mathcal{M}^{\gamma^* p}_{\lambda})^*$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\boxed{\mathcal{M}^\mu \mathcal{M}^{*\nu} = \sum_{k=1}^{9} \mathcal{V}_k^{\mu\nu} \left[\widetilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right] .}$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\mathcal{M}^{\mu}\mathcal{M}^{*\nu} = \sum_{k=1}^{9} \mathcal{V}_k^{\mu\nu} \left[\widetilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_{\rho}\mathcal{M}_{\sigma}^{*}\right].$$

Virtual-photon polarization states:

$$T^{\mu} = \epsilon_0^{\mu}, \quad X^{\mu} = -\frac{1}{\sqrt{2}}\left(\epsilon_+^{\mu} - \epsilon_-^{\mu}\right), \quad Y^{\mu} = -\frac{i}{\sqrt{2}}\left(\epsilon_+^{\mu} + \epsilon_-^{\mu}\right).$$

L polarized

T polarized
(positive/negative helicity)

$$\begin{aligned}
\mathcal{V}_1^{\mu\nu} &= X^{\mu}X^{\nu} + Y^{\mu}Y^{\nu}, \\
\mathcal{V}_2^{\mu\nu} &= T^{\mu}T^{\nu}, \\
\mathcal{V}_3^{\mu\nu} &= T^{\mu}X^{\nu} + X^{\mu}T^{\nu}, \\
\mathcal{V}_4^{\mu\nu} &= X^{\mu}X^{\nu} - Y^{\mu}Y^{\nu}, \\
\mathcal{V}_5^{\mu\nu} &= i\left(T^{\mu}X^{\nu} - X^{\mu}T^{\nu}\right), \\
\mathcal{V}_6^{\mu\nu} &= i\left(X^{\mu}Y^{\nu} - Y^{\mu}X^{\nu}\right), \\
\mathcal{V}_7^{\mu\nu} &= i\left(T^{\mu}Y^{\nu} - Y^{\mu}T^{\nu}\right), \\
\mathcal{V}_8^{\mu\nu} &= T^{\mu}Y^{\nu} + Y^{\mu}T^{\nu}, \\
\mathcal{V}_9^{\mu\nu} &= X^{\mu}Y^{\nu} + Y^{\mu}X^{\nu}.
\end{aligned}$$

Linearly independent tensor structures

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\mathcal{M}^{\mu}\mathcal{M}^{*\nu} = \sum_{k=1}^{9} \mathcal{V}_k^{\mu\nu} \left[\widetilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_{\rho}\mathcal{M}_{\sigma}^{*} \right] .$$

Virtual-photon polarization states:

$$T^{\mu} = \epsilon_0^{\mu} , \quad X^{\mu} = -\frac{1}{\sqrt{2}} \left(\epsilon_+^{\mu} - \epsilon_-^{\mu} \right) , \quad Y^{\mu} = -\frac{i}{\sqrt{2}} \left(\epsilon_+^{\mu} + \epsilon_-^{\mu} \right) .$$

L polarized

T polarized
(positive/negative helicity)

$$\begin{aligned}
\widetilde{\mathcal{V}}_1^{\mu\nu} &= \frac{1}{2} \left(X^{\mu}X^{\nu} + Y^{\mu}Y^{\nu} \right) , \\
\widetilde{\mathcal{V}}_2^{\mu\nu} &= T^{\mu}T^{\nu} , \\
\widetilde{\mathcal{V}}_3^{\mu\nu} &= -\frac{1}{2} \left(T^{\mu}X^{\nu} + X^{\mu}T^{\nu} \right) , \\
\widetilde{\mathcal{V}}_4^{\mu\nu} &= \frac{1}{2} \left(X^{\mu}X^{\nu} - Y^{\mu}Y^{\nu} \right) , \\
\widetilde{\mathcal{V}}_5^{\mu\nu} &= \frac{i}{2} \left(T^{\mu}X^{\nu} - X^{\mu}T^{\nu} \right) , \\
\widetilde{\mathcal{V}}_6^{\mu\nu} &= -\frac{i}{2} \left(X^{\mu}Y^{\nu} - Y^{\mu}X^{\nu} \right) , \\
\widetilde{\mathcal{V}}_7^{\mu\nu} &= \frac{i}{2} \left(T^{\mu}Y^{\nu} - Y^{\mu}T^{\nu} \right) , \\
\widetilde{\mathcal{V}}_8^{\mu\nu} &= -\frac{1}{2} \left(T^{\mu}Y^{\nu} + Y^{\mu}T^{\nu} \right) , \\
\widetilde{\mathcal{V}}_9^{\mu\nu} &= \frac{1}{2} \left(X^{\mu}Y^{\nu} + Y^{\mu}X^{\nu} \right) .
\end{aligned}$$

Linearly independent Inverse tensor structures

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

Using the hadronic decomposition, we can write our cross-section as:

$$d\sigma = \frac{\mathcal{C}}{2s} \sum_{k=1}^{9} \left(L_{\mu\nu}\, \mathcal{V}_k^{\mu\nu}\right) \left(\widetilde{\mathcal{V}}_k^{\rho\sigma}\, \mathcal{M}_\rho \mathcal{M}_\sigma^*\right).$$

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$$\boxed{d\sigma = \frac{\mathcal{C}}{2s}\sum_{k=1}^{9}\left(L_{\mu\nu}\,\mathcal{V}_k^{\mu\nu}\right)\left(\widetilde{\mathcal{V}}_k^{\rho\sigma}\,\mathcal{M}_\rho\mathcal{M}_\sigma^*\right).}$$

$$L_{\mu\nu}\mathcal{V}_1^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * 1$$

$$L_{\mu\nu}\mathcal{V}_2^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon$$

$$L_{\mu\nu}\mathcal{V}_3^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * (-1)\sqrt{2\epsilon(1+\epsilon)} * \cos\phi$$

$$L_{\mu\nu}\mathcal{V}_4^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon * \cos 2\phi$$

$$L_{\mu\nu}\mathcal{V}_8^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * (-1)\sqrt{2\epsilon(1+\epsilon)} * \sin\phi$$

$$L_{\mu\nu}\mathcal{V}_9^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon * \sin 2\phi$$

Contraction of leptonic tensor with linearly-independent tensors yields distinct angular modulations

$$y = \frac{P\cdot q}{P\cdot l}$$

$$\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$$

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Contraction of hadronic tensor with linearly-independent Inverse tensors

$$\widetilde{\mathcal{V}}_1^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = \frac{1}{2}\left(|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2\right)$$

Transversely-polarized photon cross-section

$$\widetilde{\mathcal{V}}_2^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = |\mathcal{M}_{Vp',0+}|^2$$

$$\widetilde{\mathcal{V}}_3^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Re}[\mathcal{M}_{Vp',0+}(\mathcal{M}^*_{Vp',-+} - \mathcal{M}^*_{Vp',++})]$$

$$\widetilde{\mathcal{V}}_4^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\mathrm{Re}[\mathcal{M}_{Vp',++}\mathcal{M}^*_{Vp',-+}]$$

$$\widetilde{\mathcal{V}}_8^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Im}[\mathcal{M}^*_{Vp',0+}(\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})]$$

$$\widetilde{\mathcal{V}}_9^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = \mathrm{Im}[\mathcal{M}_{Vp',++}\mathcal{M}^*_{Vp',-+}]$$

$\mathcal{M}_{Vp',\gamma^*p}$

V = meson polarization

p' = outgoing proton helicity

γ^* = photon polarization

p = incoming proton helicity

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

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Contraction of hadronic tensor with linearly-independent Inverse tensors

$$\widetilde{\mathcal{V}}_1^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = \frac{1}{2}(|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2)$$

$$\widetilde{\mathcal{V}}_2^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = |\mathcal{M}_{Vp',0+}|^2$$

Longitudinally-polarized photon cross-section

$$\widetilde{\mathcal{V}}_3^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Re}[\mathcal{M}_{Vp',0+}(\mathcal{M}_{Vp',-+}^* - \mathcal{M}_{Vp',++}^*)]$$

$$\widetilde{\mathcal{V}}_4^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\mathrm{Re}[\mathcal{M}_{Vp',++}\mathcal{M}_{Vp',-+}^*]$$

$$\widetilde{\mathcal{V}}_8^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Im}[\mathcal{M}_{Vp',0+}^*(\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})]$$

$$\widetilde{\mathcal{V}}_9^{\mu\nu}\mathcal{M}_\mu\mathcal{M}_\nu^* = \mathrm{Im}[\mathcal{M}_{Vp',++}\mathcal{M}_{Vp',-+}^*]$$

$\mathcal{M}_{Vp',\gamma^* p}$

V = meson polarization

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Probing gluon OAM & spin-orbit correlations via vector-mesons

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Contraction of hadronic tensor with linearly-independent Inverse tensors

$$\widetilde{\mathcal{V}}_1^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \frac{1}{2}(|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2)$$

$$\widetilde{\mathcal{V}}_2^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = |\mathcal{M}_{Vp',0+}|^2$$

$$\widetilde{\mathcal{V}}_3^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Re}[\mathcal{M}_{Vp',0+}(\mathcal{M}^*_{Vp',-+} - \mathcal{M}^*_{Vp',++})]$$

$$\widetilde{\mathcal{V}}_4^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\mathrm{Re}[\mathcal{M}_{Vp',++}\mathcal{M}^*_{Vp',-+}]$$

$$\widetilde{\mathcal{V}}_8^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}}\mathrm{Im}[\mathcal{M}^*_{Vp',0+}(\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})]$$

$$\widetilde{\mathcal{V}}_9^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \mathrm{Im}[\mathcal{M}_{Vp',++}\mathcal{M}^*_{Vp',-+}]$$

Interference terms between different photon polarizations

$\mathcal{M}_{Vp',\gamma^* p}$

$V =$ meson polarization

$p' =$ outgoing proton helicity

$\gamma^* =$ photon polarization

$p =$ incoming proton helicity

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$\frac{d\sigma}{dy dQ^2 dt d\phi} = \frac{\alpha_{\rm em}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \Bigg[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \, \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \sin\phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \, \frac{d\sigma_{\sin 2\phi}}{dt} \Bigg]$$

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$\frac{d\sigma}{dy\,dQ^2\,dt\,d\phi} = \frac{\alpha_{\rm em}}{2\pi^2}\frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2}\left[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi\frac{d\sigma_{LT}}{dt} + \epsilon\cos 2\phi\frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)}\sin\phi\frac{d\sigma_{\sin\phi}}{dt} + \epsilon\sin 2\phi\frac{d\sigma_{\sin 2\phi}}{dt}\right]$$

Remarks:

1) For definite circular polarization of the meson, only one term survives:

$$\frac{d\sigma_T^+}{dt} = \frac{x_B^2}{32\pi Q^4}C^2\left\{\delta_{\lambda,\lambda'}\left|\mathcal{H}_{eff}^{TT}\right|^2 + \delta_{\lambda,-\lambda'}\left|\vec{\Delta}_\perp\right|^2\left|\widetilde{\mathcal{E}}^{TT}\right|^2\right\}$$

$$\frac{d\sigma_T^-}{dt} = \frac{x_B^2}{32\pi Q^4}C^2\left\{\delta_{\lambda,\lambda'}\left|\mathcal{H}_{eff}^{TT}\right|^2 + \delta_{\lambda,-\lambda'}\left|\vec{\Delta}_\perp\right|^2\left|\widetilde{\mathcal{E}}^{TT}\right|^2\right\}$$

With transversely-polarized mesons, the cross section is only sensitive to GPDs

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$\frac{d\sigma}{dy dQ^2 dt d\phi} = \frac{\alpha_{\rm em}}{2\pi^2}\frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2}\Bigg[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi\frac{d\sigma_{LT}}{dt} + \epsilon\cos 2\phi\,\frac{d\sigma_{TT}}{dt}$$
$$+ \sqrt{2\epsilon(1+\epsilon)}\sin\phi\frac{d\sigma_{\sin\phi}}{dt} + \epsilon\sin 2\phi\,\frac{d\sigma_{\sin 2\phi}}{dt}\Bigg]$$

Remarks:

2) **For the longitudinally polarized vector meson case, the cross section can be expressed in terms of GTMDs**.
In general, all structure functions contribute; here, we present two representative examples.

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{\text{em}}}{2\pi^2}\frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2}\left[\frac{d\sigma_T}{dt}+\epsilon\frac{d\sigma_L}{dt}+\sqrt{2\epsilon(1+\epsilon)}\cos\phi\frac{d\sigma_{LT}}{dt}+\epsilon\cos 2\phi\frac{d\sigma_{TT}}{dt}\right.$$
$$\left.+\sqrt{2\epsilon(1+\epsilon)}\sin\phi\frac{d\sigma_{\sin\phi}}{dt}+\epsilon\sin 2\phi\frac{d\sigma_{\sin 2\phi}}{dt}\right]$$

Remarks:

Gluon OAM

2) Examples of longitudinally-polarized vector meson cross section:

$$\frac{d\sigma^0_{TT}}{dt} = -\frac{x_B^2}{16\pi Q^4}C^2\left\{\delta_{\lambda,\lambda'}\left[-\frac{1}{2}|\vec{\Delta}_\perp|^2\left|\mathcal{H}^{TL}_{eff}+\mathcal{F}^{TL}_{1,1}+\boxed{\mathcal{G}^{TL}_{1,1}}\right|^2+\frac{1}{2}|\vec{\Delta}_\perp|^2\left|\boxed{\mathcal{F}^{TL}_{1,4}}-\mathcal{H}'^{TL}_{eff}-\mathcal{G}^{TL}_{1,4}\right|^2\right]\right\},$$

$$\frac{d\sigma^0_{\sin 2\phi}}{dt} = \frac{x_B^2}{16\pi Q^4}C^2\left\{-\delta_{\lambda,\lambda'}\lambda|\vec{\Delta}_\perp|^2\,\text{Re}\left[\left(\mathcal{H}^{TL}_{eff}+\mathcal{F}^{TL}_{1,1}+\boxed{\mathcal{G}^{TL}_{1,1}}\right)\left(\boxed{\mathcal{F}^{TL*}_{1,4}}-\mathcal{H}'^{TL*}_{eff}-\mathcal{G}^{TL*}_{1,4}\right)\right]\right\}.$$

Gluon's spin-orbit correlations

Probing gluon OAM & spin-orbit correlations via vector-mesons

Cross section

$$\frac{d\sigma}{dy\,dQ^2\,dt\,d\phi} = \frac{\alpha_{\text{em}}}{2\pi^2}\frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2}\left[\frac{d\sigma_T}{dt}+\epsilon\frac{d\sigma_L}{dt}+\sqrt{2\epsilon(1+\epsilon)}\cos\phi\frac{d\sigma_{LT}}{dt}+\epsilon\cos 2\phi\frac{d\sigma_{TT}}{dt}\right.$$
$$\left.+\sqrt{2\epsilon(1+\epsilon)}\sin\phi\frac{d\sigma_{\sin\phi}}{dt}+\epsilon\sin 2\phi\frac{d\sigma_{\sin 2\phi}}{dt}\right]$$

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The **cos 2ϕ** modulation stands out as a particularly striking signal: it can be measured with an <u>unpolarized proton</u>. Thus, this signal provides a clean laboratory signal for probing the gluon OAM and gluon spin–orbit correlations.

Axial-vector production

Axial-vector production

Remarks:

- For the vector meson results, we assumed that the quark anti-quark pair shared an equal amount of the meson’s momentum (z=1/2).

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Axial-vector production

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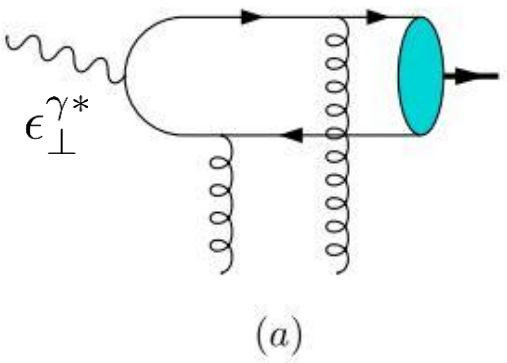
- For the vector meson results, we assumed that the quark anti-quark pair shared an equal amount of the meson's momentum (z=1/2). However, for the axial-vector case, both the twist-2 and twist-3 contributions vanish at the symmetric point z = 1/2. Thus, obtaining a nonvanishing result requires working away from this point.

Example: Diagram "a"



Notation: $\mathcal{M}^{\gamma^*, M}_{twist}$

$$\mathcal{H}^{T,L}_{t3,k_\perp} = -\frac{8im_{AV}\left(m_{AV}^2-Q^2\right)\xi^2(1-2z)^2\epsilon_\perp^{ij}(\epsilon_\perp^{\gamma*i}k_\perp^j)}{\left(m_{AV}^2(2x(z-1)+\xi(4z^2-6z+1))+2Q^2(z-1)(\xi+x)\right)\left(m_{AV}^2(2xz+\xi(4z^2-2z-1))+2Q^2z(x-\xi)\right)}$$

Outline

- Generalized TMDs & connection to spin physics
- Observable(s) for quark/gluon OAM & spin-orbit correlations:
  1) Exclusive pseudoscalar meson &
  2) Exclusive heavy (axial-) vector meson production
- Summary

Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics

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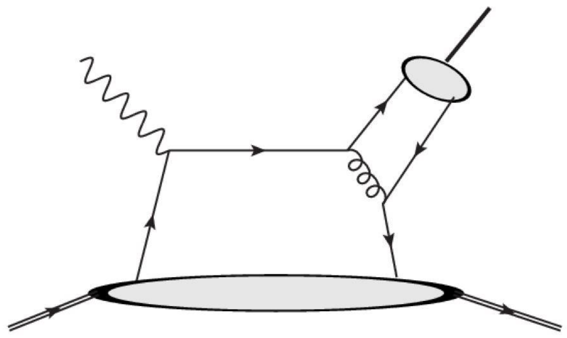
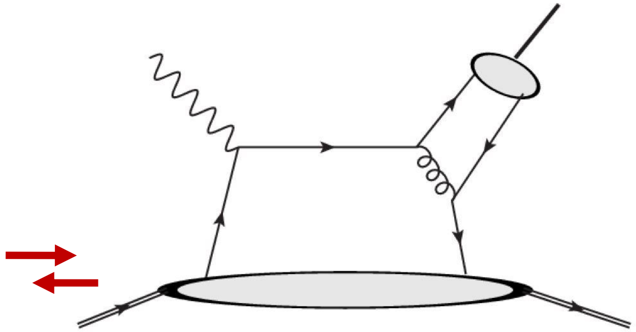
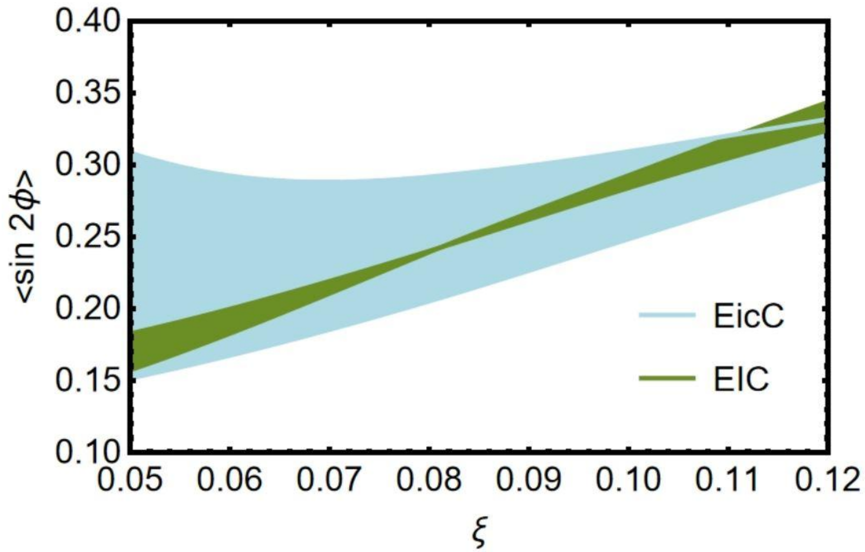
- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive π^0 production in ep collisions



Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive π^0 production in ep collisions





- Asymmetry is substantial & thus exclusive π^0 production in ep collisions maybe a promising route to constrain quark OAM

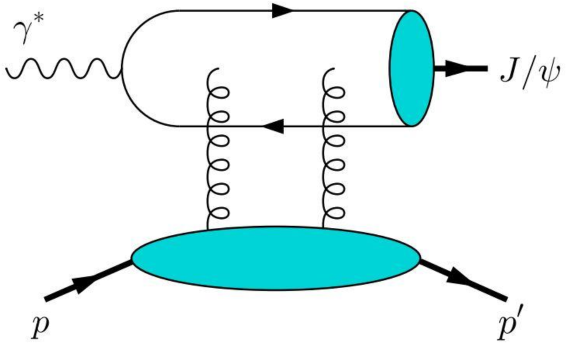
$$\frac{d\sigma}{dt\,dQ^2\,dx_B\,d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right]\right.$$

$$\left.\boxed{+\lambda \sin(2\phi)\, 2a\, \mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}^*_{1,1}+\mathcal{G}^*_{1,1}\right)\right]}\right\}$$

Summary

- Probe **gluon OAM and spin-orbit correlations** via exclusive heavy (axial-) vector production



$$\frac{d\sigma}{dy\,dQ^2\,dt\,d\phi} = \frac{\alpha_{\rm em}}{2\pi^2}\frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2}\left[\frac{d\sigma_T}{dt}+\epsilon\frac{d\sigma_L}{dt}+\sqrt{2\epsilon(1+\epsilon)}\cos\phi\frac{d\sigma_{LT}}{dt}+\epsilon\cos 2\phi\frac{d\sigma_{TT}}{dt}+\sqrt{2\epsilon(1+\epsilon)}\sin\phi\frac{d\sigma_{\sin\phi}}{dt}+\epsilon\sin 2\phi\frac{d\sigma_{\sin 2\phi}}{dt}\right]$$

$$\frac{d\sigma^0_{TT}}{dt} = -\frac{x_B^2}{16\pi Q^4}C^2\left\{\delta_{\lambda,\lambda'}\left[-\frac{1}{2}|\vec{\Delta}_\perp|^2\left|\mathcal{H}^{TL}_{eff}+\mathcal{F}^{TL}_{1,1}+\boxed{\mathcal{G}^{TL}_{1,1}}\right|^2+\frac{1}{2}|\vec{\Delta}_\perp|^2\left|\boxed{\mathcal{F}^{TL}_{1,4}}-\mathcal{H}'^{TL}_{eff}-\mathcal{G}^{TL}_{1,4}\right|^2\right]\right\}$$

The cos 2ϕ modulation stands out as a striking signal: it can be measured with an unpolarized proton

- Rich prospects with CLAS data

Backup slides

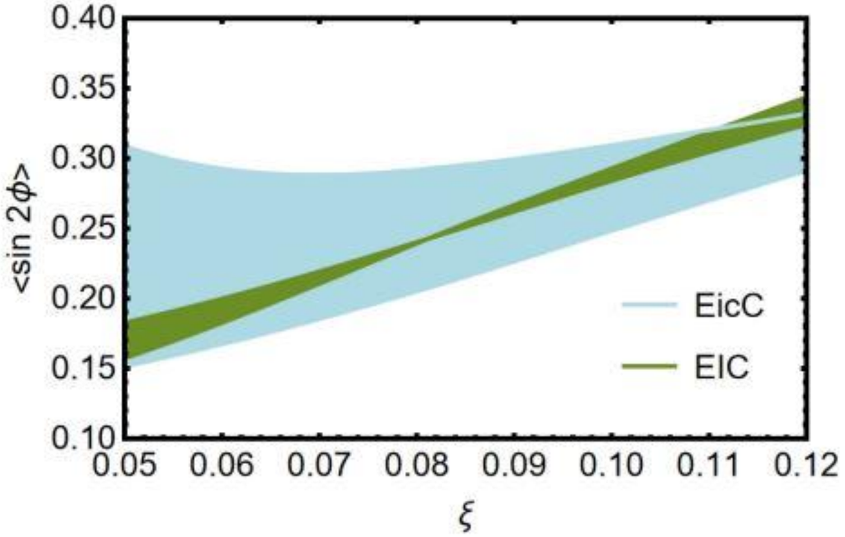


FIG. 3: The unpolarized cross section, as given by Eq. (15), is displayed in the top plot for EIC kinematics with $Q^2 = 10\,\mathrm{GeV}^2$ and $\sqrt{s_{ep}} = 100\,\mathrm{GeV}$, as well as for EicC kinematics with $Q^2 = 3\,\mathrm{GeV}^2$ and $\sqrt{s_{ep}} = 16\,\mathrm{GeV}$. The unpolarized cross section for the EIC case is re-scaled by a factor of 100. The bottom plot shows the average value of $\langle \sin(2\phi) \rangle$ given by Eq. (16). The variable t is integrated over the range $[-0.5\,\mathrm{GeV}^2, -\frac{4\xi^2 M^2}{1-\xi^2}]$.The error bands are obtained by varying the value of $\sqrt{\langle p_\perp^2 \rangle}$ from 150 MeV to 250 MeV and the value of α', which determines the t-dependence of the various distributions in the double distribution approach (see supplementary material), from 1.2 to 1.4.

We notice that other fitting for $g_{1T}^{(1)}(x)$ exist too [82]. Meanwhile, the $k_\perp$ moment of $F_{1,2}$ can be related to the Qiu-Sterman function,

$$\int d^2k_\perp \frac{k_\perp^2}{M} \mathrm{Im}[F_{1,2}(x, \xi = 0, \Delta_\perp = 0, k_\perp)]$$
$$= -\int d^2k_\perp \frac{k_\perp^2}{M} f_{1T}^\perp(x, k_\perp) = T_F(x, x) \tag{6}$$

where the Qiu-Sterman function is parametrized as [83],

$$T_F(x, x) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q}(1-x)^{\beta_q} q(x) \tag{7}$$

with $\alpha_u = 1.051, \alpha_d = 1.552$, $\beta_u = \beta_d = 4.857$, and $N_u = 1.06, N_d = -0.163$. See also Refs. [84], [85], and [86] for the state-of-the-art extractions of the Sivers functions. Once the x-dependence of the $k_\perp$ moments of $F_{1,2}$ and $G_{1,2}$ is reconstructed as explained above, we reconstruct their (ξ, t)-dependence in accordance with the double distribution

CFFs:

$$\mathcal{H}_{eff}^{LL} = -\int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)} \frac{Q\sqrt{1-\xi^2}}{m_V} H_{eff}^g, \tag{25}$$

$$\widetilde{\mathcal{E}}^{LL} = -\int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)} \frac{Q}{m_V} \frac{1}{2M\sqrt{1-\xi^2}} E^g, \tag{26}$$

$$\mathcal{H}_{eff}^{TT} = \int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)} \sqrt{1-\xi^2} H_{eff}^g, \tag{27}$$

$$\widetilde{\mathcal{E}}^{TT} = \int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)} \frac{1}{2M\sqrt{1-\xi^2}} E^g, \tag{28}$$

$$\mathcal{H}_{eff}^{TL} = \int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2} \frac{\xi^2\sqrt{1-\xi^2}}{m_V} H_{eff}^g, \tag{29}$$

$$\mathcal{H}'^{TL}_{eff} = \int_{-1}^{1} \frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2} \frac{ix\xi\sqrt{1-\xi^2}}{m_V} H'^g_{eff}, \tag{30}$$

$$\mathcal{F}_{1,1}^{TL} = \int_{-1}^{1} \frac{dx}{(x-\xi-i\varepsilon)^2(x+\xi+i\varepsilon)^2} \frac{2x\xi}{m_V} \frac{1}{\sqrt{1-\xi^2}} \int d^2k_\perp \left(\frac{k_\perp \cdot \Delta_\perp}{\Delta_\perp^2}\right) F_{1,1}^g, \tag{31}$$

$$\mathcal{G}_{1,1}^{TL} = -\frac{1}{\sqrt{1-\xi^2}} \int_{-1}^{1} dx \frac{\xi^2}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2} \int d^2k_\perp \left(\frac{k_\perp^2}{M^2}\right) G_{1,1}^g, \tag{32}$$

$$\widetilde{\mathcal{F}}_{1,2}^{TL} = M\sqrt{1-\xi^2} \int_{-1}^{1} dx \frac{x\xi}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2} \int d^2k_\perp \left(\frac{k_\perp^2}{M^2}\right) F_{1,2}^g, \tag{33}$$

$$\widetilde{\mathcal{G}}_{1,2}^{TL} = -M\sqrt{1-\xi^2} \int_{-1}^{1} dx \frac{\xi^2}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2} \int d^2k_\perp \left(\frac{k_\perp^2}{M^2}\right) G_{1,2}^g, \tag{34}$$

$$\mathcal{F}_{1,4}^{TL} = \frac{1}{\sqrt{1-\xi^2}} \int_{-1}^{1} dx \frac{ix\xi}{m_V(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2} \int d^2k_\perp \left(\frac{k_\perp^2}{M^2}\right) F_{1,4}^g, \tag{35}$$

$$\mathcal{G}_{1,4}^{TL} = \int \frac{dx}{(x-\xi+i\varepsilon)^2(x+\xi-i\varepsilon)^2} \frac{2i\xi^2}{m_V} \frac{1}{\sqrt{1-\xi^2}} \int d^2k_\perp \left(\frac{k_\perp \cdot \Delta_\perp}{\Delta_\perp^2}\right) G_{1,4}^g. \tag{36}$$

in the lepton plane. The outgoing lepton momentum follows from momentum conservation. Upon contracting the resulting leptonic tensor with the tensor structures of the basis constructed from the photon polarization vectors, as described above, we obtain the following distinct angular modulations:

$$L_{\mu\nu}\,\mathcal{V}_3^{\mu\nu} \propto \cos\phi, \quad L_{\mu\nu}\,\mathcal{V}_4^{\mu\nu} \propto \cos 2\phi,$$
$$L_{\mu\nu}\,\mathcal{V}_8^{\mu\nu} \propto \sin\phi, \quad L_{\mu\nu}\,\mathcal{V}_9^{\mu\nu} \propto \sin 2\phi. \tag{66}$$

Here, ϕ may equivalently be identified as the angle between the lepton scattering plane and the hadron production plane provided we choose $\Delta_\perp = (|\vec{\Delta}_\perp|, 0)$, which we adopt to keep the expressions simple (and for this reason we henceforth drop the subscript $l_\perp$ from the angle). These angular modulations are well known in the literature [45–47] and originate from specific interference patterns: the $\cos\phi$ and $\sin\phi$ terms arise from the interference between twist-2 and twist-3 amplitudes, while the $\cos 2\phi$ and $\sin 2\phi$ terms arise from the interference between twist-3 amplitudes.

Next, we work, without loss of generality, with an initial proton of positive helicity and proceed to calculate the second term in parentheses of Eq. (61). Below, the photon-helicity amplitudes will be denoted by $\mathcal{M}_{Vp',\gamma^* p}$. It then follows straightforwardly that the differential cross section for exclusive vector–meson electroproduction can be written as

$$\frac{d\sigma}{dy\,dQ^2\,dt\,d\phi} = \frac{\alpha_{\rm em}}{2\pi^2}\frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2}\Bigg[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi\,\frac{d\sigma_{LT}}{dt} + \epsilon\cos 2\phi\,\frac{d\sigma_{TT}}{dt}$$
$$+ \sqrt{2\epsilon(1+\epsilon)}\sin\phi\,\frac{d\sigma_{\sin\phi}}{dt} + \epsilon\sin 2\phi\,\frac{d\sigma_{\sin 2\phi}}{dt}\Bigg] \tag{67}$$

where the individual structure functions are expressed as

$$\frac{d\sigma_T}{dt} = \frac{x_B^2}{32\pi Q^4}\left(|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2\right), \tag{68}$$

$$\frac{d\sigma_L}{dt} = \frac{x_B^2}{16\pi Q^4}|\mathcal{M}_{Vp',0+}|^2, \tag{69}$$

$$\frac{d\sigma_{LT}}{dt} = \frac{\sqrt{2}x_B^2}{32\pi Q^4}\,\mathrm{Re}\big[\mathcal{M}_{Vp',0+}\left(\mathcal{M}^*_{Vp',-+} - \mathcal{M}^*_{Vp',++}\right)\big], \tag{70}$$

$$\frac{d\sigma_{TT}}{dt} = -\frac{x_B^2}{16\pi Q^4}\,\mathrm{Re}\big[\mathcal{M}_{Vp',++}\,\mathcal{M}^*_{Vp',-+}\big], \tag{71}$$

$$\frac{d\sigma_{\sin\phi}}{dt} = \frac{\sqrt{2}x_B^2}{32\pi Q^4}\,\mathrm{Im}\big[\mathcal{M}^*_{Vp',0+}\left(\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++}\right)\big], \tag{72}$$

$$\frac{d\sigma_{\sin 2\phi}}{dt} = \frac{x_B^2}{16\pi Q^4}\,\mathrm{Im}\big[\mathcal{M}_{Vp',++}\,\mathcal{M}^*_{Vp',-+}\big]. \tag{73}$$

Here we have expressed our results in terms of the virtual-photon flux parameter ϵ, defined as, $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$. The meson with positive circular polarization ($V=+$), the results read:

$$\boxed{\frac{d\sigma_T^+}{dt} = \frac{x_B^2}{32\pi Q^4}\, C^2 \left\{ \delta_{\lambda,\lambda'} \left|\mathcal{H}^{TT}_{eff}\right|^2 + \delta_{\lambda,-\lambda'} \left|\vec{\Delta}_\perp\right|^2 \left|\widetilde{\mathcal{E}}^{TT}\right|^2 \right\},} \tag{74}$$

and vanishing in all other cases. Similarly, in the case of a vector meson with negative circular polarization ($V=-$), the results read:

$$\boxed{\frac{d\sigma_T^-}{dt} = \frac{x_B^2}{32\pi Q^4}\, C^2 \left\{ \delta_{\lambda,\lambda'} \left|\mathcal{H}^{TT}_{eff}\right|^2 + \delta_{\lambda,-\lambda'} \left|\vec{\Delta}_\perp\right|^2 \left|\widetilde{\mathcal{E}}^{TT}\right|^2 \right\},} \tag{75}$$

and vanishing in all other cases. Both of the above results arise from the interference between twist-2 contributions.

Finally, for longitudinally polarized vector meson ($V=0$), the results read:

$$\frac{d\sigma_T^0}{dt} = \frac{x_B^2}{32\pi Q^4}\, C^2 \Big\{ \delta_{\lambda,\lambda'} \Big[|\vec{\Delta}_\perp|^2 \left|\mathcal{H}^{TL}_{eff} + \mathcal{F}^{TL}_{1,1} + \mathcal{G}^{TL}_{1,1}\right|^2 + |\vec{\Delta}_\perp|^2 \left|\mathcal{F}^{TL}_{1,4} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}^{TL}_{1,4}\right|^2 \Big] + 2\,\delta_{\lambda,-\lambda'} \left|\widetilde{\mathcal{F}}^{TL}_{1,2} - \widetilde{\mathcal{G}}^{TL}_{1,2}\right|^2 \Big\}, \tag{76}$$

$$\frac{d\sigma_L^0}{dt} = \frac{x_B^2}{16\pi Q^4}\, C^2 \Big\{ \delta_{\lambda,\lambda'} \left|\mathcal{H}^{LL}_{eff}\right|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 \left|\widetilde{\mathcal{E}}^{LL}\right|^2 \Big\}, \tag{77}$$

$$\frac{d\sigma_{LT}^0}{dt} = \frac{\sqrt{2}\,x_B^2}{32\pi Q^4}\, C^2 \Big\{ \delta_{\lambda,\lambda'} \sqrt{2}|\vec{\Delta}_\perp|\, \mathrm{Re}\Big[\mathcal{H}^{LL}_{eff} \left(\mathcal{H}^{TL*}_{eff} + \mathcal{F}^{TL*}_{1,1} + \mathcal{G}^{TL*}_{1,1}\right)\Big] + \delta_{\lambda,-\lambda'} \sqrt{2}|\vec{\Delta}_\perp|\, \mathrm{Re}\Big[\widetilde{\mathcal{E}}^{LL} \left(\widetilde{\mathcal{F}}^{TL*}_{1,2} - \widetilde{\mathcal{G}}^{TL*}_{1,2}\right)\Big] \Big\}, \tag{78}$$

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4}\, C^2 \Big\{ \delta_{\lambda,\lambda'} \Big[-\frac{1}{2}|\vec{\Delta}_\perp|^2 \left|\mathcal{H}^{TL}_{eff} + \mathcal{F}^{TL}_{1,1} + \mathcal{G}^{TL}_{1,1}\right|^2 + \frac{1}{2}|\vec{\Delta}_\perp|^2 \left|\mathcal{F}^{TL}_{1,4} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}^{TL}_{1,4}\right|^2 \Big] \Big\}, \tag{79}$$

$$\frac{d\sigma_{\sin\phi}^0}{dt} = \frac{\sqrt{2}\,x_B^2}{32\pi Q^4}\, C^2 \Big\{ -\delta_{\lambda,\lambda'}\, \lambda |\vec{\Delta}_\perp|\, \mathrm{Re}\Big[\sqrt{2}\,\mathcal{H}^{LL*}_{eff} \left(\mathcal{F}^{TL}_{1,4} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}^{TL}_{1,4}\right)\Big] - \lambda\, \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|\, \mathrm{Im}\Big[\sqrt{2}\,\widetilde{\mathcal{E}}^{LL*} \left(\widetilde{\mathcal{F}}^{TL}_{1,2} - \widetilde{\mathcal{G}}^{TL}_{1,2}\right)\Big] \Big\}, \tag{80}$$

$$\frac{d\sigma_{\sin2\phi}^0}{dt} = \frac{x_B^2}{16\pi Q^4}\, C^2 \Big\{ -\delta_{\lambda,\lambda'}\, \lambda |\vec{\Delta}_\perp|^2\, \mathrm{Re}\Big[\left(\mathcal{H}^{TL}_{eff} + \mathcal{F}^{TL}_{1,1} + \mathcal{G}^{TL}_{1,1}\right) \left(\mathcal{F}^{TL*}_{1,4} - \mathcal{H}'^{TL*}_{eff} - \mathcal{G}^{TL*}_{1,4}\right)\Big] \Big\}. \tag{81}$$

Twist nature:

Here, $d\sigma_L^0/dt$ arises from twist-2 × twist-2 contributions, while the $d\sigma_{LT}^0/dt$ and $d\sigma_{\sin\phi}^0/dt$ terms result from the interference between twist-2 and twist-3 contributions. All remaining terms involve purely twist-3 × twist-3 contributions. From these expressions, we conclude that the polarization-independent $\cos 2\phi$ and polarization-dependent $\sin 2\phi$ terms provide exceptionally clean signatures of the gluon GTMDs $F_{1,4}$ and $G_{1,1}$. While other observables, such as $d\sigma_T/dt$

Final cross section: longitudinal meson

$$\frac{d\sigma_T^0}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \Bigg\{ \delta_{\lambda,\lambda'} \Big[|\vec{\Delta}_\perp|^2 \, |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + |\vec{\Delta}_\perp|^2 \Big| \mathcal{F}_{1,4}^{TL} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}_{1,4}^{TL} \Big|^2 \Big] + 2\, \delta_{\lambda,-\lambda'} \Big| \widetilde{\mathcal{F}}_{1,2}^{TL} - \widetilde{\mathcal{G}}_{1,2}^{TL} \Big|^2 \Bigg\}$$

$$\frac{d\sigma_L^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \Bigg\{ \delta_{\lambda,\lambda'} \, |\mathcal{H}_{eff}^{LL}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 \Big| \widetilde{\mathcal{E}}^{LL} \Big|^2 \Bigg\}$$

$$\frac{d\sigma_{LT}^0}{dt} = \frac{\sqrt{2}\, x_B^2}{32\pi Q^4} C^2 \Bigg\{ \delta_{\lambda,\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \, \mathrm{Re}\Big[\mathcal{H}_{eff}^{LL} \left(\mathcal{H}_{eff}^{TL*} + \mathcal{F}_{1,1}^{TL*} + \mathcal{G}_{1,1}^{TL*} \right) \Big] \delta_{\lambda,-\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \, \mathrm{Re}\Big[\widetilde{\mathcal{E}}^{LL} \left(\widetilde{\mathcal{F}}_{1,2}^{TL*} - \widetilde{\mathcal{G}}_{1,2}^{TL*} \right) \Big] \Bigg\}$$

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \Bigg\{ \delta_{\lambda,\lambda'} \Big[-\frac{1}{2} |\vec{\Delta}_\perp|^2 \, |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + \frac{1}{2} |\vec{\Delta}_\perp|^2 \Big| \mathcal{F}_{1,4}^{TL} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}_{1,4}^{TL} \Big|^2 \Big] \Bigg\}$$

$$\frac{d\sigma_{\sin\phi}^0}{dt} = \frac{\sqrt{2}\, x_B^2}{32\pi Q^4} C^2 \Bigg\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp| \, \mathrm{Re}\Big[\sqrt{2}\, \mathcal{H}_{eff}^{LL*} \left(\mathcal{F}_{1,4}^{TL} - \mathcal{H}'^{TL}_{eff} - \mathcal{G}_{1,4}^{TL} \right) \Big] - \lambda\, \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp| \, \mathrm{Im}\Big[\sqrt{2}\, \widetilde{\mathcal{E}}^{LL*} \left(\widetilde{\mathcal{F}}_{1,2}^{TL} - \widetilde{\mathcal{G}}_{1,2}^{TL} \right) \Big] \Bigg\}$$

$$\frac{d\sigma_{\sin 2\phi}^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \Bigg\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp|^2 \, \mathrm{Re}\Big[\left(\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL} \right) \left(\mathcal{F}_{1,4}^{TL*} - \mathcal{H}'^{TL*}_{eff} - \mathcal{G}_{1,4}^{TL*} \right) \Big] \Bigg\}$$

First ever results for heavy vector DVMP cross section in terms of GTMDs