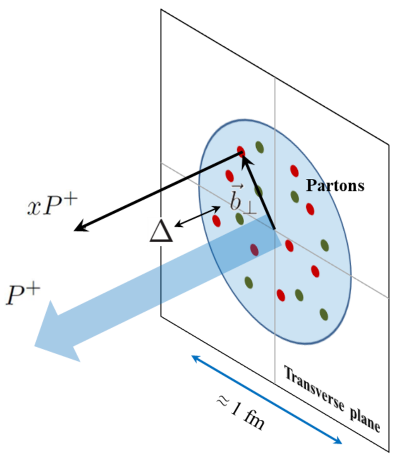
CNF lecture series, July13-17



Hadron structure with GPDs (1/5)

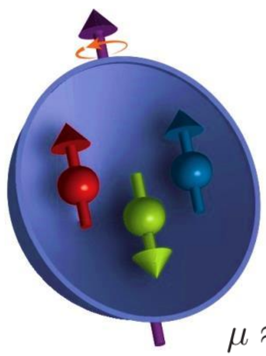
Cédric Lorcé

Nucleon structure

Non-relativistic picture

dominated by constituents

Spectroscopy



$\mu \approx \Lambda_{\mathrm{QCD}}$

Mass

$$M_N c^2 \sim \sum_Q M_Q c^2 + E_{\mathrm{binding}}$$

~ 102 % ~ -2 %

Spin

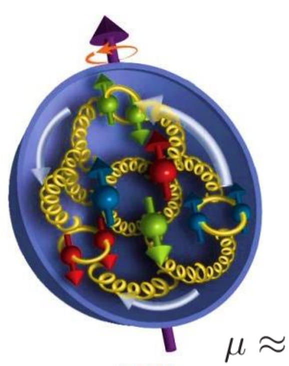
$$J_z^N \sim \sum_Q S_z^Q$$

~ 100 %

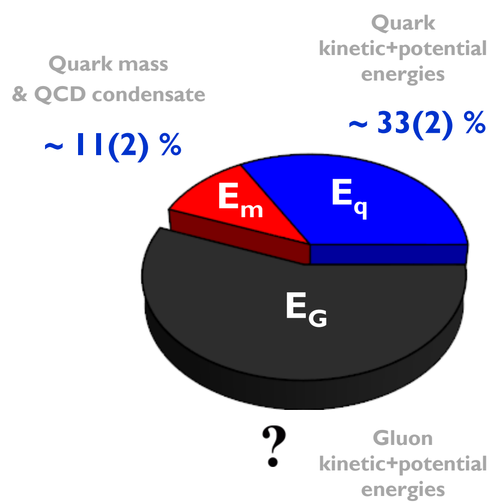
Relativistic picture

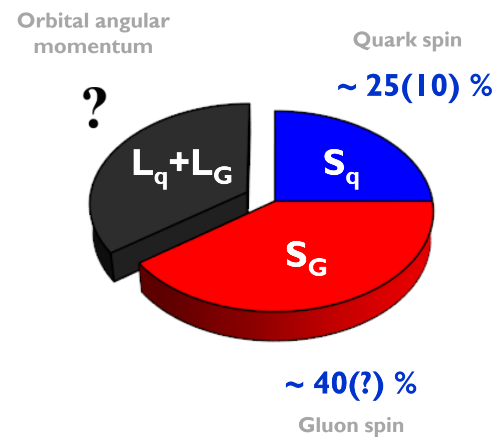
dominated by dynamics

High-energy scattering

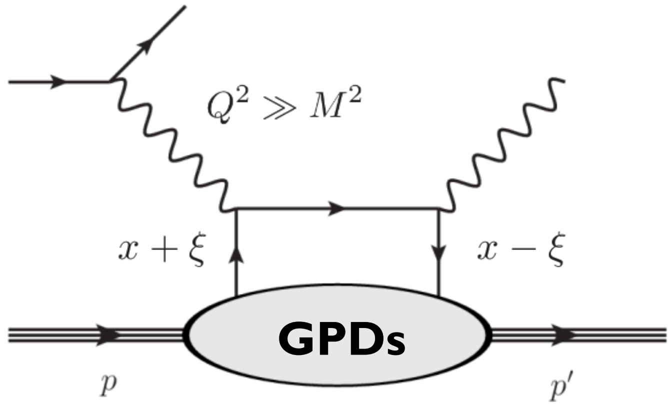


$\mu \approx 2\,\mathrm{GeV}$





Objective of these lectures



Deeply virtual Compton scattering (DVCS)

$$\Downarrow$$

Off-forward matrix element

$$\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \underbrace{\langle p', s'|}_{\text{Off-forward matrix element}} \underbrace{\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})}_{\text{Non-local operator}} \underbrace{|p, s\rangle}_{\text{Off-forward matrix element}}\Big|_{z^+=|\vec{z}_\perp|=0}$$

Non-local operator

What is the physical meaning/content of such a correlator ?

Outline

Mo	• Spatial imaging
Tu-We	• Light-front & phase-space pictures
We	• Energy-momentum tensor
Th-Fr	• Mass & spin decompositions
Fr	• Mechanical properties

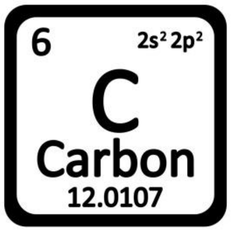
→ ***Do not hesitate to ask questions !***

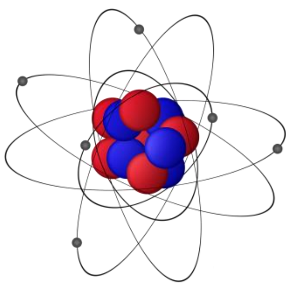
Spatial imaging

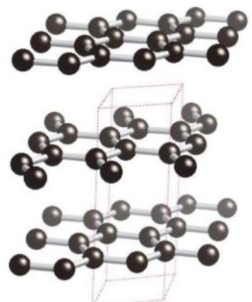
Spatial structure

3D structure is fundamental to understand physical properties

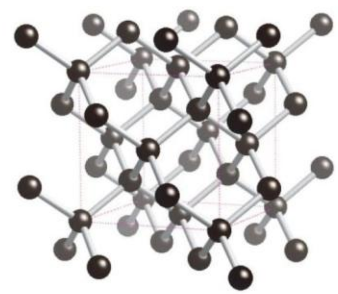
i.e. thermal, electrical, mechanical, …



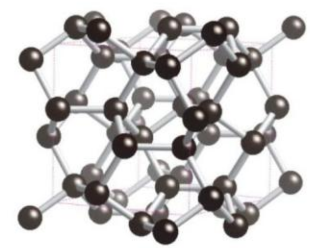




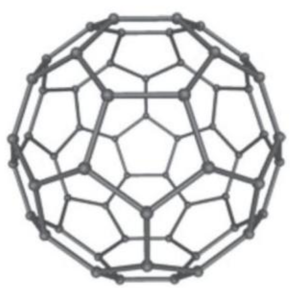
graphite



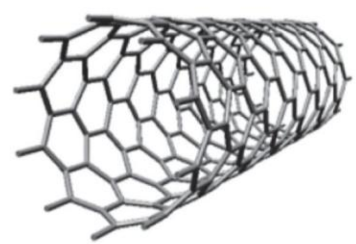
diamond



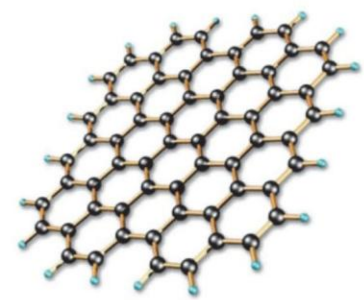
BC8



fullerene



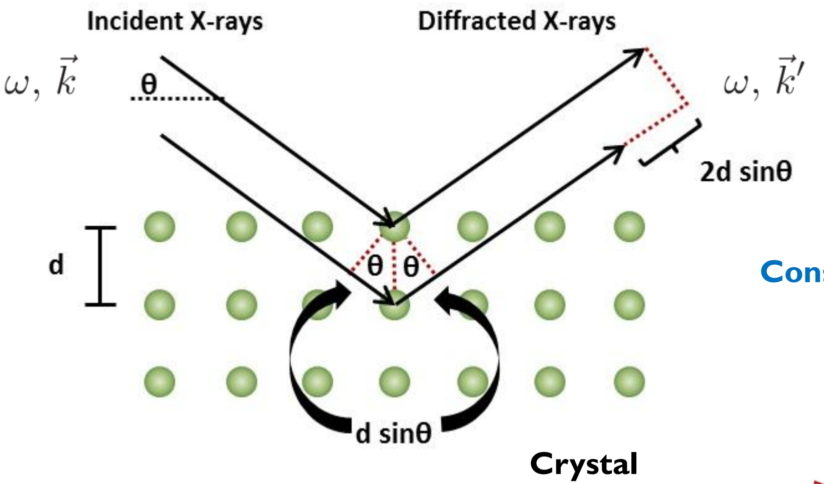
nanotube



graphene

Spatial structure through elastic scattering

<u>Example:</u> X-ray diffraction

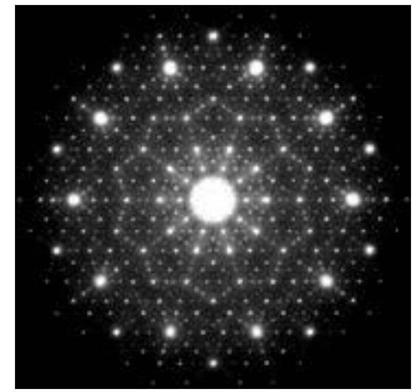


Constructive interference
(Bragg's law)

$$2d\,\sin\theta = n\lambda$$

$$\boxed{\lambda \sim d}$$

Diffraction pattern



$\propto |A_{\rm scatt}|^2$

Scattered amplitude

$$A_{\rm scatt} \propto F(\vec{q}) = \int \mathrm{d}^3r\, e^{i\vec{q}\cdot\vec{r}}\,\rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

Form factor

Scatterer distribution

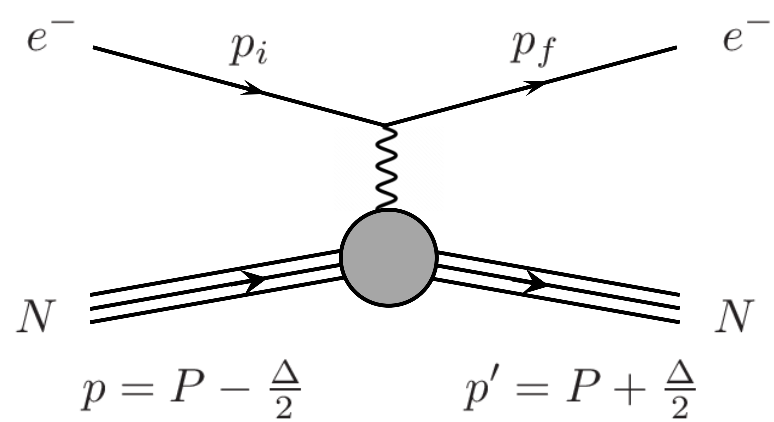
Nuclear elastic scattering

Crystals & atoms $d \approx 10^{-10}\,\mathrm{m} \quad \Rightarrow \quad \hbar\omega \approx 10^{4}\,\mathrm{eV}$ → **X-rays**

Nuclei & nucleons $d \approx 10^{-15}\,\mathrm{m} \quad \Rightarrow \quad \hbar\omega \approx 10^{9}\,\mathrm{eV}$ → **High-energy electron beams**

Large recoil for light nuclei!

Relativistic treatment
in Born approximation



Spin-0 target

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\text{pointlike}} = [F(Q^2)]^2$$

Spin-1/2 target

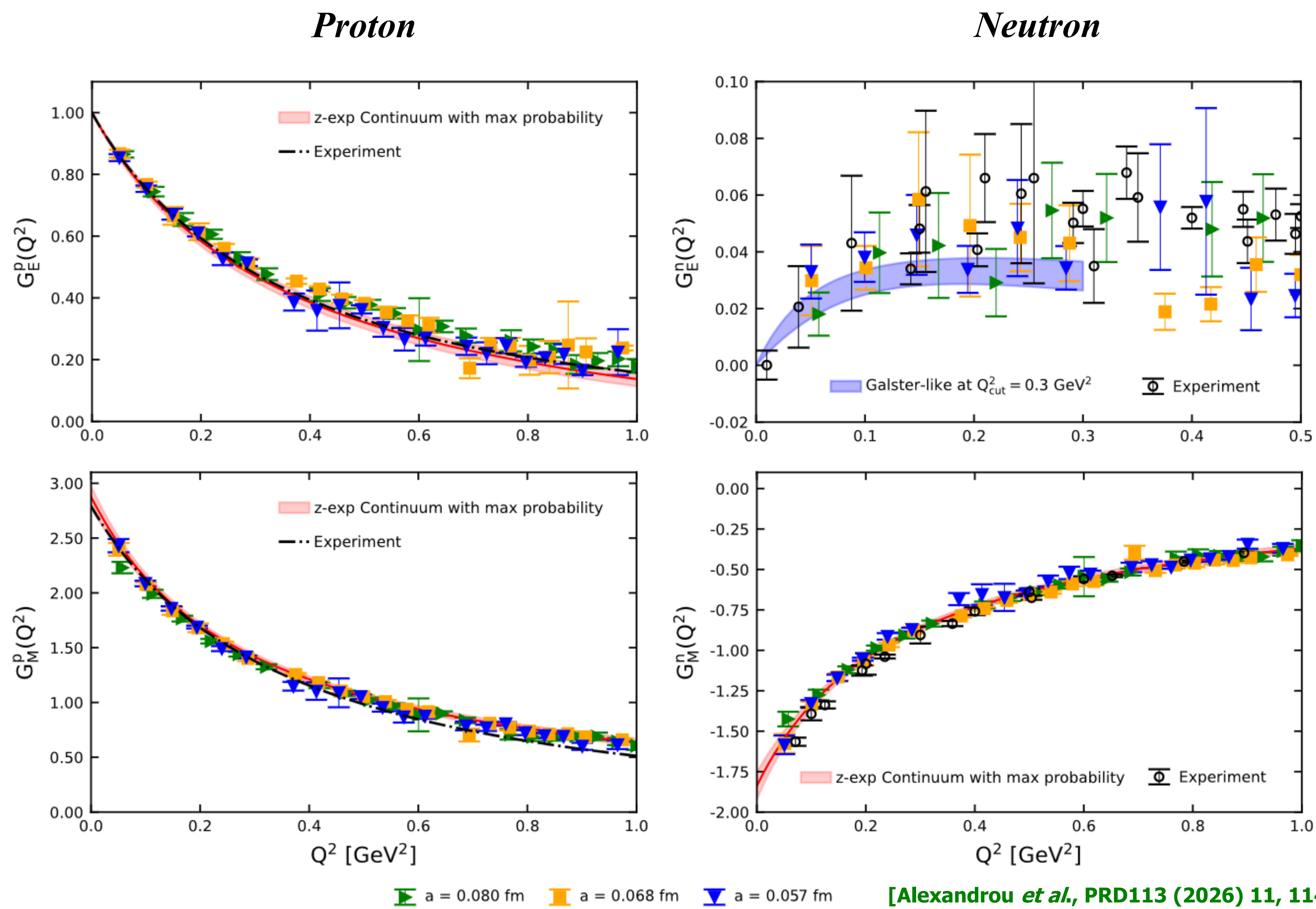
$$= \left\{ \left[G_E(Q^2)\right]^2 + \frac{\tau}{\epsilon}\left[G_M(Q^2)\right]^2 \right\} \frac{1}{1+\tau}$$

Electric form factor (G_E) **Magnetic form factor** (G_M)

$$Q^2 = -\Delta^2$$
$$\tau = Q^2/4M_N^2$$
$$\epsilon = (1 + 2(1+\tau)\tan^2\tfrac{\theta_e}{2})^{-1}$$

[Rosenbluth, PR79 (1950) 615]
[Hofstadter, RMP28 (1956) 214]
[Yennie, Lévy, Ravenhall, RMP29 (1957) 144]

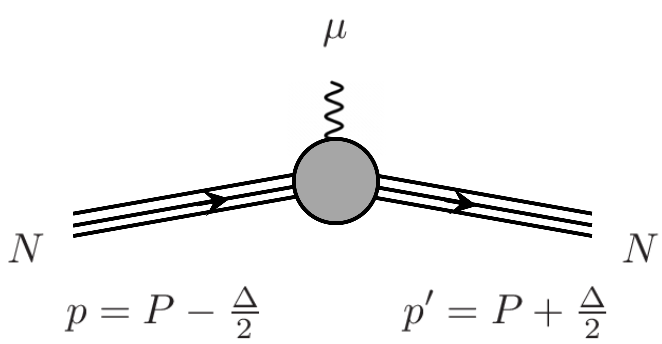
Nucleon form factors (low Q^2)



[Alexandrou *et al.*, PRD113 (2026) 11, 114524]

Electromagnetic current matrix elements

Normalization $\langle p'|p\rangle = (2\pi)^3 2p^0\,\delta^{(3)}(\vec{p}\,'-\vec{p})$



$$\langle p', s'|J^\mu(0)|p, s\rangle = \overline{u}(p', s')\Gamma^\mu(P,\Delta)u(p,s)$$

$$\Gamma^\mu(P,\Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N}F_2(Q^2)$$

Dirac form factor — **Pauli form factor**

$F_1(0) = q_N,$ $\quad F_2(0) = \kappa_N$

Electric charge — **Anomalous magnetic moment**

Sachs form factors

$Q^2 = -\Delta^2$

$\tau = Q^2/4M_N^2$

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Mass shell constraints

$$(P \pm \tfrac{\Delta}{2})^2 = M_N^2 \quad \Rightarrow \quad \boxed{\begin{aligned} &P\cdot\Delta = 0 \\ &P^2 = M_N^2(1+\tau)\end{aligned}}$$

[Foldy, PR87 (1952) 688]
[Ernst, Sachs, Wali, PR119 (1960) 1105]
[Sachs, PR126 (1962) 2256]

Non-relativistic interpretation

Localized states $\boxed{|\vec{r}\rangle = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\, |\vec{p}\rangle}$

Normalization $\langle\vec{p}\,'|\vec{p}\rangle = (2\pi)^3\,\delta^{(3)}(\vec{p}\,'-\vec{p})$

$$\langle\vec{r}\,'|\rho(\vec{x})|\vec{r}\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{i\vec{P}\cdot(\vec{r}\,'-\vec{r})}\, e^{-i\vec{\Delta}\cdot(\vec{x}-\frac{\vec{r}\,'+\vec{r}}{2})}\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|\rho(\vec{0})|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

Charge density

Galilean symmetry $\boxed{\langle\vec{P}+\frac{\vec{\Delta}}{2}|\rho(\vec{0})|\vec{P}-\frac{\vec{\Delta}}{2}\rangle = \langle\frac{\vec{\Delta}}{2}|\rho(\vec{0})|-\frac{\vec{\Delta}}{2}\rangle}$

Non-relativistic boost
$\vec{p}\mapsto\vec{p}+M_N\vec{v}$

$$\langle\vec{r}\,'|\rho(\vec{x})|\vec{r}\rangle = \delta^{(3)}(\vec{r}\,'-\vec{r})\,\underbrace{\int\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot(\vec{x}-\vec{r})}\,\langle\tfrac{\vec{\Delta}}{2}|\rho(\vec{0})|-\tfrac{\vec{\Delta}}{2}\rangle}_{\equiv\,\rho(\vec{x}-\vec{r})}$$

Internal distribution

Generic state $\langle\psi|\psi\rangle = 1$

Wave packet $\psi(\vec{r}) = \langle\vec{r}|\psi\rangle$

$$\Rightarrow\quad \langle\rho\rangle_\psi(\vec{x}) = \langle\psi|\rho(\vec{x})|\psi\rangle = \int \mathrm{d}^3r\,|\psi(\vec{r})|^2\,\rho(\vec{x}-\vec{r})$$

Probabilistic interpretation

Semi-relativistic interpretation (Sachs approach)

Generic state $\langle\psi|\psi\rangle = 1$

Normalization $\langle\vec{p}\,'|\vec{p}\rangle = (2\pi)^3\,\delta^{(3)}(\vec{p}\,'-\vec{p})$

Wave packet $\tilde{\psi}(\vec{p}) = \langle\vec{p}|\psi\rangle$

$$\langle\psi|O(x)|\psi\rangle = \int\frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,\tilde{\psi}^*(\vec{P}+\tfrac{\vec{\Delta}}{2})\tilde{\psi}(\vec{P}-\tfrac{\vec{\Delta}}{2})\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|O(x)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

Crucial assumption $\tilde{\psi}(\vec{P}\pm\tfrac{\vec{\Delta}}{2}) \approx \tilde{\psi}(\vec{P})$

$$\langle\psi|O(x)|\psi\rangle \overset{x^0=0}{\approx} \int\frac{\mathrm{d}^3P}{(2\pi)^3}\,|\tilde{\psi}(\vec{P})|^2\underbrace{\int\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\cdot\vec{x}}\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|O(0)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle}_{\text{Internal distribution}}$$

Probabilistic interpretation

Validity domain $1/D \ll |\vec{\Delta}| \ll |\delta\vec{p}| \ll P^0$

(System size)$^{-1}$ | Resolution | Width of wave packet | (Boosted Compton wavelength)$^{-1}$

Breit frame $|\vec{P}| = 0 \quad\Rightarrow\quad P^0 \approx M_N$

$|\tilde{\psi}(\vec{P})|^2 \to (2\pi)^3\delta^{(3)}(\vec{P})$ ⚠ **Clash with** $\tilde{\psi}(\vec{P}\pm\tfrac{\vec{\Delta}}{2}) \approx \tilde{\psi}(\vec{P})$ **!**

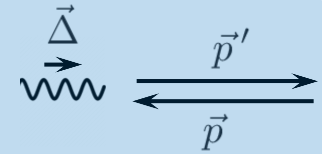
Hydrogen $M_H D_H \approx 10^5$ ✓

Nucleon $M_N D_N \approx 4$ ⚠

[Sachs, PR126 (1962) 2256]
[Burkardt, PRD62 (2000) 071503]
[Belitsky, Ji, Yuan, PRD69 (2004) 074014]

Semi-relativistic interpretation (Sachs approach)

Breit (aka brick-wall) **frame**



$$\vec{P} = \vec{0} \quad \Rightarrow \quad \Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} = 0$$

$$\langle p', s'|J^0(0)|p, s\rangle\big|_{\mathrm{BF}} = 2M_N \delta_{s's}\, G_E(Q^2)$$

$$\langle p', s'|\vec{J}(0)|p, s\rangle\big|_{\mathrm{BF}} = i(\vec{\sigma}_{s's} \times \vec{\Delta})\, G_M(Q^2)$$

Same structure as in non-relativistic case !

$$Q^2\big|_{\mathrm{BF}} = \vec{\Delta}^2$$

3D charge distribution

$$\boxed{\rho_E^{\mathrm{BF}}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{G_E(Q^2)}{\sqrt{1+\tau}}}$$

Relativistic recoil corrections ?

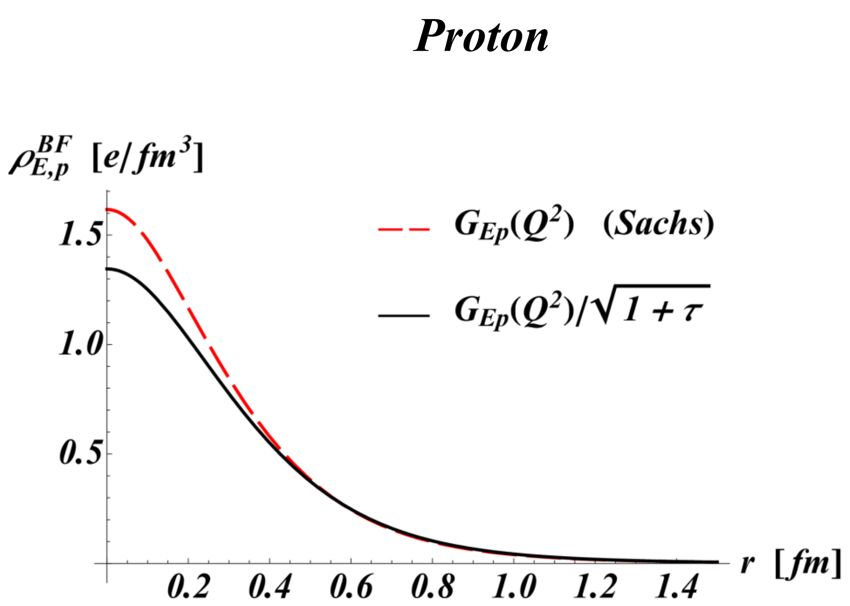
$$P^0\big|_{\mathrm{BF}} = M_N\sqrt{1+\tau}$$

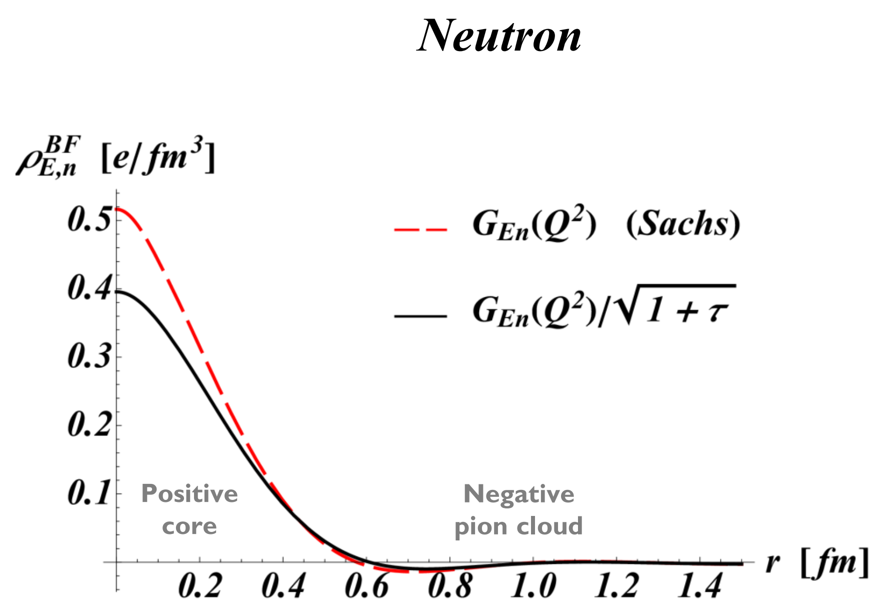
responsible for the Darwin term
in the non-relativistic expansion

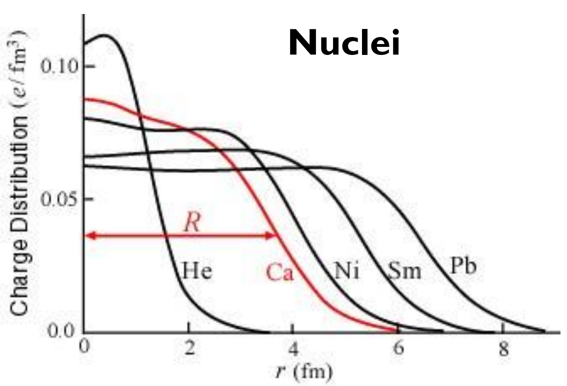
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big/\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\mathrm{pointlike}} = \left\{\left[G_E(Q^2)\right]^2 + \frac{\tau}{\epsilon}\left[G_M(Q^2)\right]^2\right\}\frac{1}{1+\tau}$$

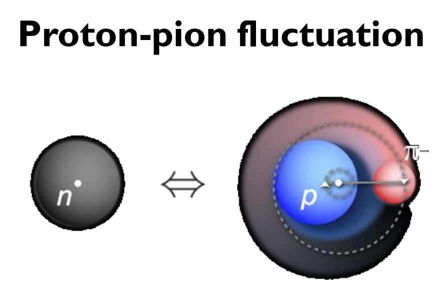
[Sachs, PR126 (1962) 2256]
[Friar, Negele, In *Adv. Nucl. Phys., Vol.8* (1975) 219]

Breit frame distributions









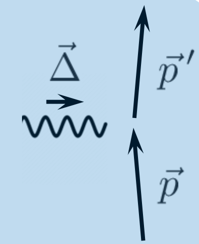
[C.L., PRL125 (2020) 232002]

Relativistic interpretation (IMF approach)

Probabilistic interpretation

Validity domain $1/D \ll |\vec{\Delta}| \ll |\delta\vec{p}| \ll P^0$

Infinite-momentum frame



$$P_z \to \infty \quad \Rightarrow \quad \Delta^0 \approx \Delta_z \ll P^0$$

$$\langle p', \lambda' | J^0(0) | p, \lambda \rangle \big|_{\text{IMF}} = 2P^0 \left[\delta_{\lambda'\lambda}\, F_1(Q^2) + \frac{i(\vec{\sigma}_{\lambda'\lambda} \times \vec{\Delta})_z}{2M_N}\, F_2(Q^2) \right] \qquad Q^2\big|_{\text{IMF}} = \vec{\Delta}_\perp^2$$

2D charge distribution

$$\boxed{\rho_E^{\text{IMF}}(\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, F_1(Q^2) - \frac{(\vec{S} \times \vec{\nabla})_z}{M_N} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, F_2(Q^2)}$$

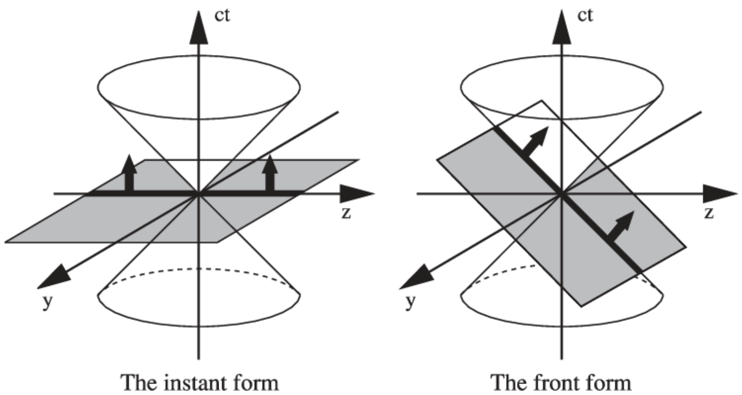
Galilean symmetry under finite boosts → **No recoil correction !**

[Bouchiat, Fayet, Meyer, NPB34 (1971) 157]
[Soper, PRD15 (1977) 1141]
[Burkardt, PRD62 (2000) 071503]

Other approaches with similar results

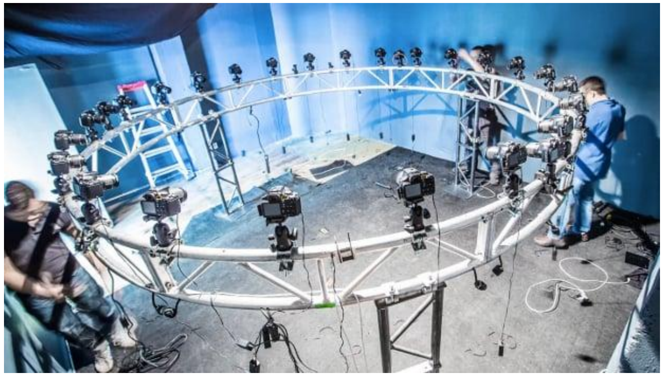
Light-front quantization and Drell-Yan frame $\Delta^+ = 0$ (no need to consider IMF)

$$a = [a^+, a^1, a^2, a^-], \quad a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$



[Ralston, Jain, Buniy, AIP Conf. Proc. 549 (2000) 1, 302]
[Burkardt, IJMPA 18 (2003) 2, 173]
[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]
[Freese, Miller, PRD105 (2022) 1, 014003]

Method of dimensional counting (IMF averaged over all directions)

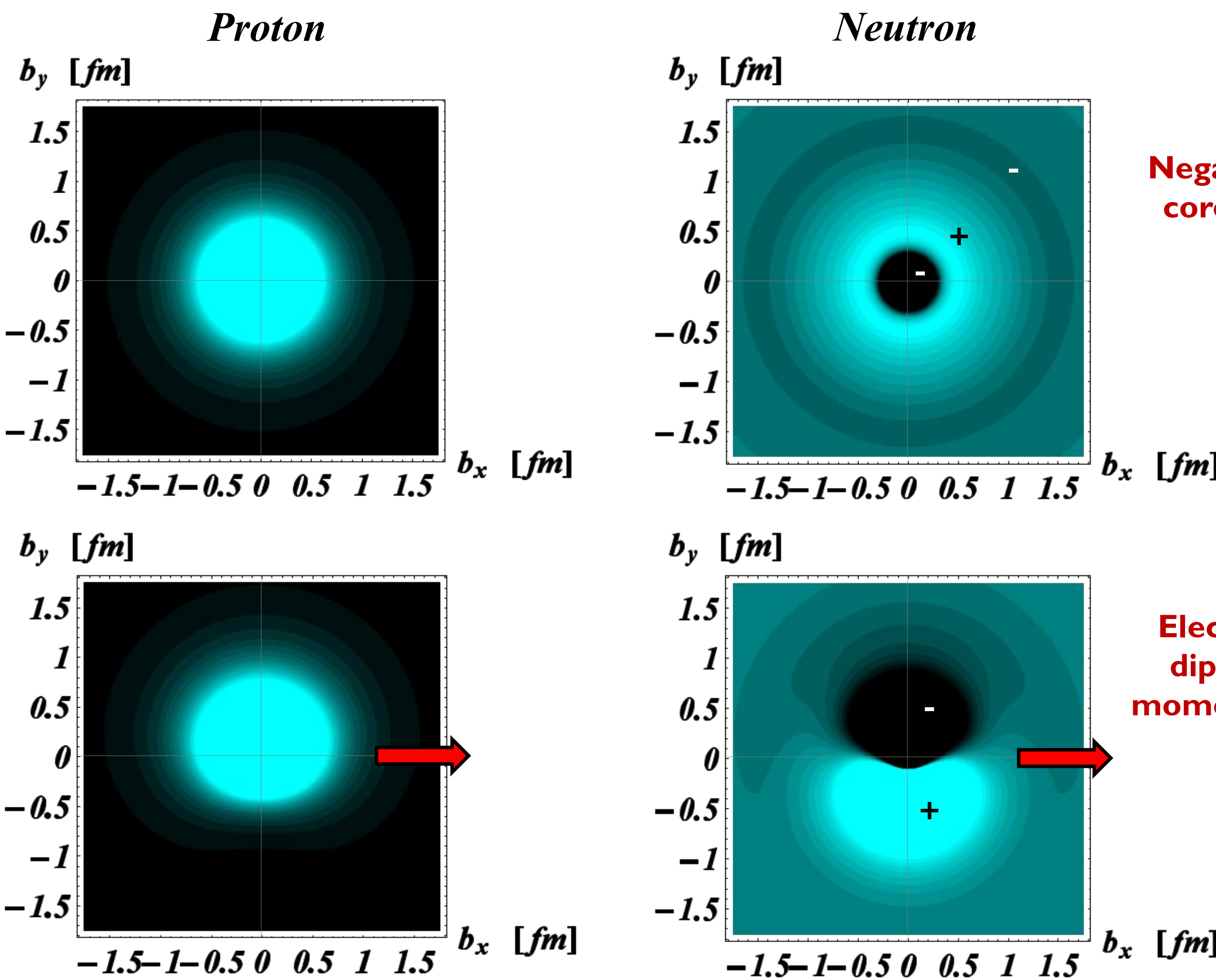


[Fleming, In *Phys. Reality & Math. Descrip.* (1974) 357]
[Epelbaum, Gegelia, Lange, Meissner, Polyakov, PRL129 (2022) 012001]
[Panteleeva, Epelbaum, Gegelia, Meissner, PRD106 (2022) 5, 056019]

IMF distributions

$$\vec{S} = \frac{\hbar}{2}\,\vec{e}_z$$

$$\vec{S} = \frac{\hbar}{2}\,\vec{e}_x$$

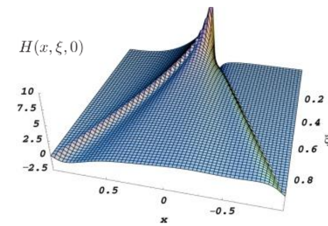


Negative core !?!

Electric dipole moment !?!

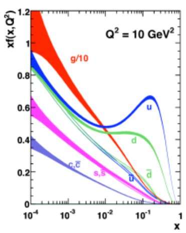
[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

Generalized parton distributions



GPDs $\langle p'|O(x)|p\rangle$

x, Δ



$\langle p|O(x)|p\rangle$

Non-local operator

PDFs

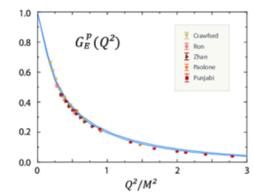
x

FFs

Δ

$\langle p'|O|p\rangle$

Off-forward matrix element



$\langle p|O|p\rangle$ **Charges**

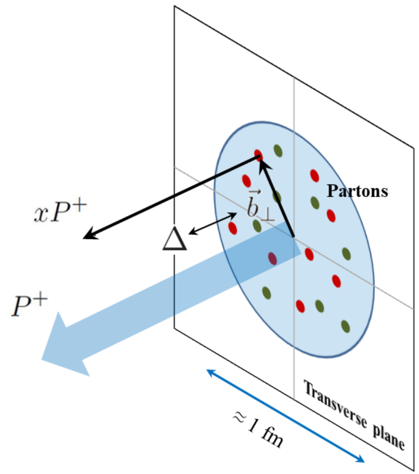
$\Delta = 0$

$\int \mathrm{d}x$

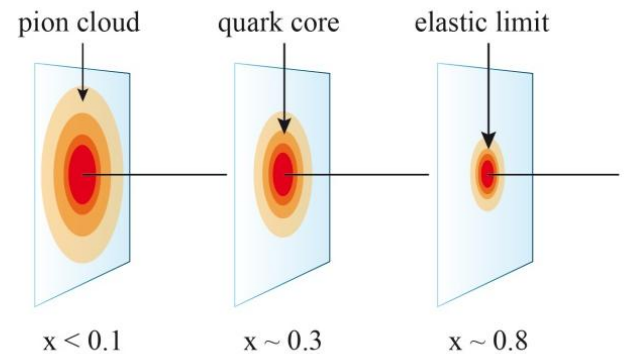
$$O(x) \sim \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z}{2}) \cdots \psi(\tfrac{z}{2})\big|_{z^+=|\vec{z}_\perp|=0} \quad \xrightarrow{\int \mathrm{d}x} \quad O \sim \overline{\psi}(0) \cdots \psi(0)$$

Hadron tomography

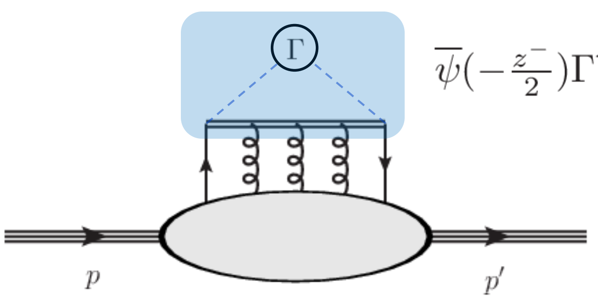
(2+1)D picture $\Delta^+ = 0$



tomos **= « cut/section »**
graphein **= « to draw/write »**



Gauge invariance and spin



$$\overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2})$$

$$\mathcal{W} = \mathcal{P}\left[\exp\left(ig\int_{\frac{z^-}{2}}^{-\frac{z^-}{2}} \mathrm{d}y^-\, A^+(y)\right)\right] \overset{A^+=0}{\Rightarrow} 1$$

Leading-twist Γ	$\gamma^+ \sim \delta_{\lambda'_q\lambda_q}$	**Unpolarized quark**
	$\gamma^+\gamma_5 \sim (\sigma_3)_{\lambda'_q\lambda_q}$	**Longitudinally polarized quark**
	$i\sigma^{j+}\gamma_5 \sim (\sigma_j)_{\lambda'_q\lambda_q}$	**Tranversely polarized quark**

[Soper, PRD15 (1977) 1141]
[Burkardt, PRD62 (2000) 071503]
[Burkardt, IJMPA18 (2003) 2, 173]
[Diehl, Hägler, EPJC44 (2005) 87]