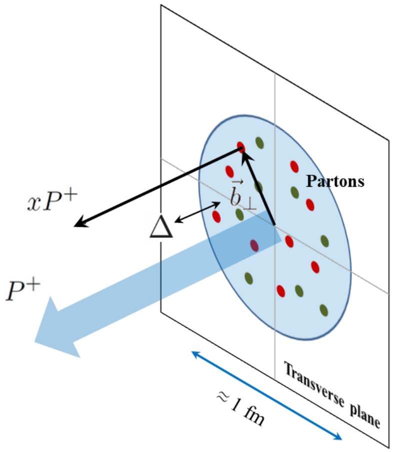
CNF lecture series, July 13-17



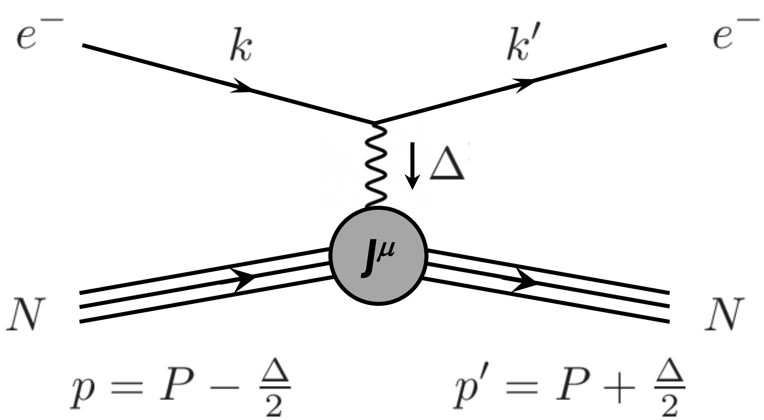
Hadron structure with GPDs (3/5)

Cédric Lorcé

Energy-momentum tensor

Local probes

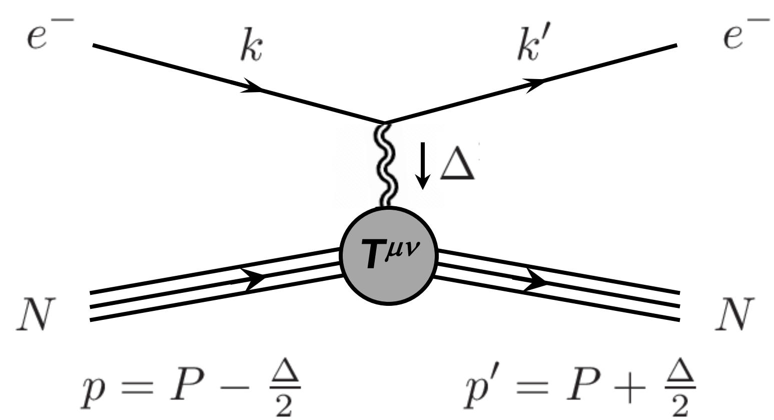
Photon exchange
(~1)



Electromagnetic current

Electromagnetic form factors

Graviton exchange
($\sim 10^{-36}$)

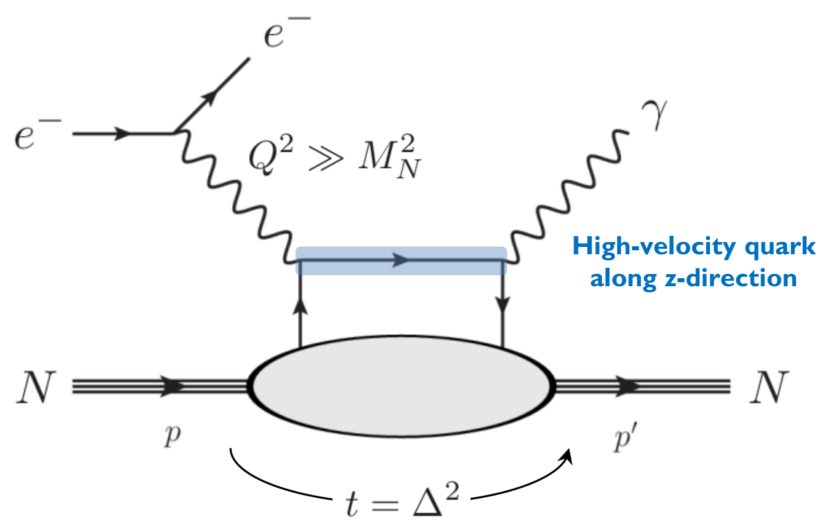


Energy-momentum tensor

Gravitational form factors

A non-local probe

Deeply virtual Compton scattering (DVCS)



$$\overline{\psi}(-\tfrac{z^-}{2})\gamma^+\mathcal{W}\psi(\tfrac{z^-}{2}) \quad \approx \quad \overline{\psi}(0)\gamma^+\psi(0) \quad + \quad z^-\,\overline{\psi}(0)\gamma^+\tfrac{i}{2}\overleftrightarrow{D}^+\psi(0) \quad + \quad \cdots$$

$$J^+ \propto J^0 + J^3 \qquad\qquad T^{++} \propto T^{00} + T^{03} + T^{30} + T^{33}$$

Electromagnetic current **Energy-momentum tensor**

[Ji, PRL78 (1997) 610]
[Ji, JPG24 (1998) 1181]
[Diehl, PR388 (2003) 41]
[Belitsky, Radyushkin, PR418 (2005) 1]

Mellin moments

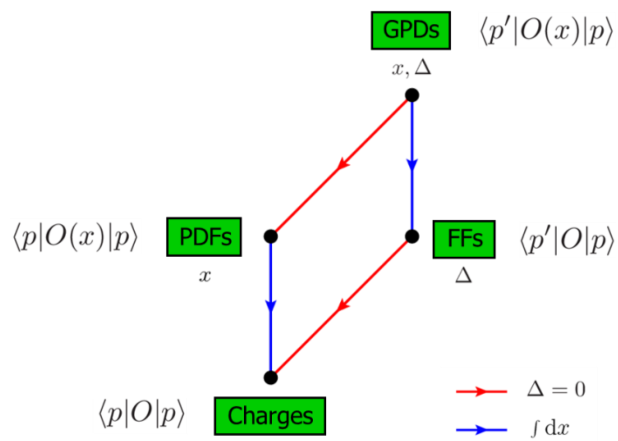
Local LF operators

$$\int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2}) = \frac{1}{(P^+)^{n+1}}\,\overline{\psi}(0)\Gamma(\tfrac{i}{2}\overleftrightarrow{D}^+)^n\psi(0)$$

First moment $\Gamma = \gamma^+$

$$\boxed{\begin{aligned}\int \mathrm{d}x\, H_q(x,\xi,t) &= F_1^q(t)\\ \int \mathrm{d}x\, E_q(x,\xi,t) &= F_2^q(t)\end{aligned}}$$

Electromagnetic form factors



Second moment $\Gamma = \gamma^+$

$$\boxed{\begin{aligned}\int \mathrm{d}x\, x\, H_q(x,\xi,t) &= A_q(t) + 4\xi^2\, C_q(t)\\ \int \mathrm{d}x\, x\, \frac{1}{2}\,[H_q + E_q](x,\xi,t) &= J_q(t)\end{aligned}}$$

Gravitational form factors

[Ji, PRL78 (1997) 610]
[Ji, JPG24 (1998) 1181]
[Diehl, PR388 (2003) 41]
[Belitsky, Radyushkin, PR418 (2005) 1]

Mellin moments

Local LF operators

$$\int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2}) = \frac{1}{(P^+)^{n+1}}\, \overline{\psi}(0)\Gamma(\tfrac{i}{2}\overleftrightarrow{D}^+)^n\psi(0)$$

Spin-dependent operators

$\Gamma = \gamma^+\gamma_5$ ➡ **Longitudinal polarization and spin-orbit correlation**

[C.L., Pasquini, PRD84 (2011) 014015]
[C.L., PLB735 (2014) 344]

$\Gamma = \gamma^i\gamma_5$ ➡ **Transverse spin and color Lorentz force**

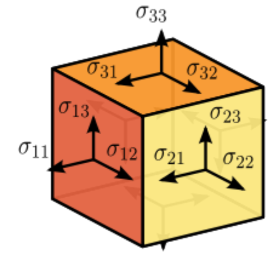
[Burkardt, PRD88 (2013) 114502]
[Aslan, Burkardt, Schlegel, PRD100 (2019) 9, 096021]

$\Gamma = \sigma^{i+}\gamma_5$ ➡ **Transverse polarization and spin-orbit correlation**

[C.L., Pasquini, PRD93 (2016) 3, 034040]
[Bhoonah, C.L., PLB774 (2017) 435]

Energy-momentum tensor (EMT)

Mass, spin and pressure are all encoded in the EMT



Energy density — Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

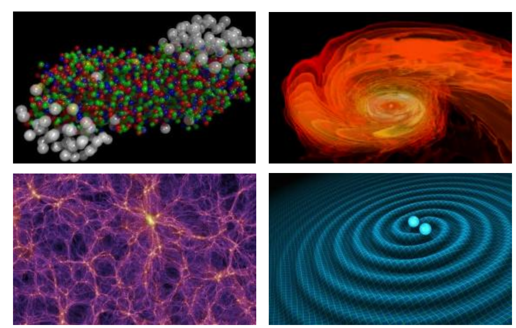
Shear stress

Normal stress (pressure)

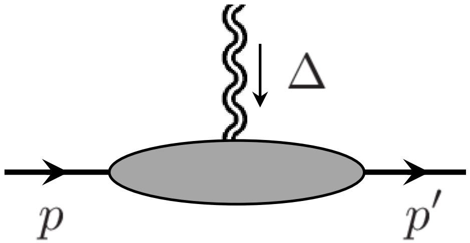
Energy flux — Momentum flux

Central object for

- Nucleon mechanical properties
- Quark-gluon plasma
- Relativistic hydrodynamics
- Stellar structure and dynamics
- Cosmology
- Gravitational waves
- Modified theories of gravitation
- …



Gravitational form factors (GFFs)



Poincaré symmetry constrains the form of the EMT matrix elements

Symmetrized variables $P = \frac{p'+p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$

$$p'^2 = p^2 = M^2 \Rightarrow \begin{cases} P \cdot \Delta = 0 \\ P^2 = M^2 - \frac{\Delta^2}{4} \end{cases}$$

Spin-0 target $T^{\mu\nu} = \sum_{a=q,g} T_a^{\mu\nu}$

$$\langle p'|T_a^{\mu\nu}(0)|p\rangle = 2M\left[\frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu}\bar{C}_a(t)\right]$$

Non-conserved

$$0 = \langle p'|\partial_\mu T^{\mu\nu}(x)|p\rangle = i\Delta_\mu \langle p'|T^{\mu\nu}(x)|p\rangle \Rightarrow \boxed{\sum_a \bar{C}_a(t) = 0}$$

[Kobzarev, Okun, JETP16 (1963) 5, 1343]
[Pagels, PR144 (1966) 4, 1250]
[Ji, PRL78 (1997) 610]

Gravitational form factors (GFFs)

Spin-1/2 target

$$\langle p', s'|T_a^{\mu\nu}(0)|p, s\rangle = \overline{u}(p', s')\Gamma_a^{\mu\nu}(P, \Delta)u(p, s)$$

$$\boxed{\begin{aligned}\Gamma_a^{\mu\nu}(P, \Delta) = {} & \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu}\bar{C}_a(t) \\ & + \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M} J_a(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{2M} S_a(t)\end{aligned}}$$

$$x^{\{\mu}y^{\nu\}} = x^\mu y^\nu + x^\nu y^\mu$$
$$x^{[\mu}y^{\nu]} = x^\mu y^\nu - x^\nu y^\mu$$

NB: Because of the Dirac equation, alternative but equivalent parametrizations may *look* quite different !

Gordon identity

$$\overline{u}(p', s')\gamma^\mu u(p, s) = \overline{u}(p', s')\left[\frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\right] u(p, s)$$

[Pagels, PR144 (1966) 4, 1250]
[Ji, PRL78 (1997) 610]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

Four-momentum conservation

Expectation value

$$\boxed{\langle P_a^\mu \rangle = \frac{\langle p|P_a^\mu|p\rangle}{\langle p|p\rangle} = \frac{\langle p|T_a^{0\mu}(0)|p\rangle}{2p^0}}$$

$$\langle p|T_a^{\mu\nu}(0)|p\rangle = 2p^\mu p^\nu A_a(0) + 2M^2 g^{\mu\nu}\bar{C}_a(0)$$

$$\Rightarrow \quad \langle P_a^\mu \rangle = p^\mu A_a(0) + \frac{M^2}{p^0}\, g^{0\mu}\bar{C}_a(0)$$

Not a four-vector !
(unless state is massless)

Light-front version

$$\langle P_{a,\mathrm{LF}}^\mu \rangle = p^\mu A_a(0) + \frac{M^2}{p^+}\, g^{+\mu}\bar{C}_a(0) \quad \underset{g^{++}=0}{\Rightarrow} \quad \langle P_{a,\mathrm{LF}}^+ \rangle = \underbrace{A_a(0)}_{=\int \mathrm{d}x\, x\, f_1^a(x)}\, p^+$$

$p^\pm = (p^0 \pm p^3)/\sqrt{2}$

Deep-inelastic scattering

Four-momentum sum rules

$$p^\mu = \sum_a \langle P_a^\mu \rangle \quad \Rightarrow \quad \boxed{\begin{aligned} \sum_a A_a(0) &= 1 \\ \sum_a \bar{C}_a(0) &= 0 \end{aligned}}$$

Why two sum rules ?

What is the meaning of $\bar{C}_a(0)$?

Mechanical equilibrium

Physical interpretation is simpler in target rest frame

$$\frac{\langle p_\text{rest}|\int \mathrm{d}^3x\, T_a^{\mu\nu}(x)|p_\text{rest}\rangle}{\langle p_\text{rest}|p_\text{rest}\rangle} = M \left(\begin{array}{c|ccc} A_a(0)+\bar{C}_a(0) & 0 & 0 & 0 \\ \hline 0 & -\bar{C}_a(0) & 0 & 0 \\ 0 & 0 & -\bar{C}_a(0) & 0 \\ 0 & 0 & 0 & -\bar{C}_a(0) \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{c|ccc} \varepsilon_a & 0 & 0 & 0 \\ \hline 0 & p_a & 0 & 0 \\ 0 & 0 & p_a & 0 \\ 0 & 0 & 0 & p_a \end{array}\right) V$$

➡ $-\bar{C}_a(0)$ **measures the average stress (or pressure) exerted by subsystem** a

Mechanical equilibrium implies $\sum_a p_a = 0 \quad \Rightarrow \quad \sum_a \bar{C}_a(0) = 0$

Virial theorem