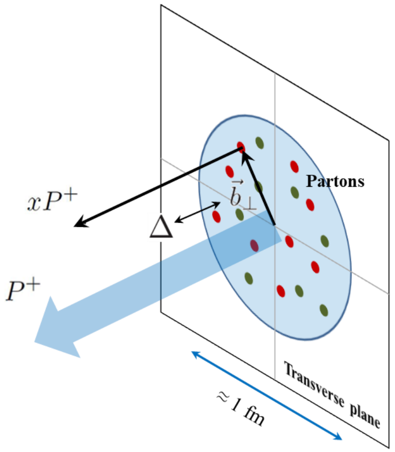
CNF lecture series, July13-17



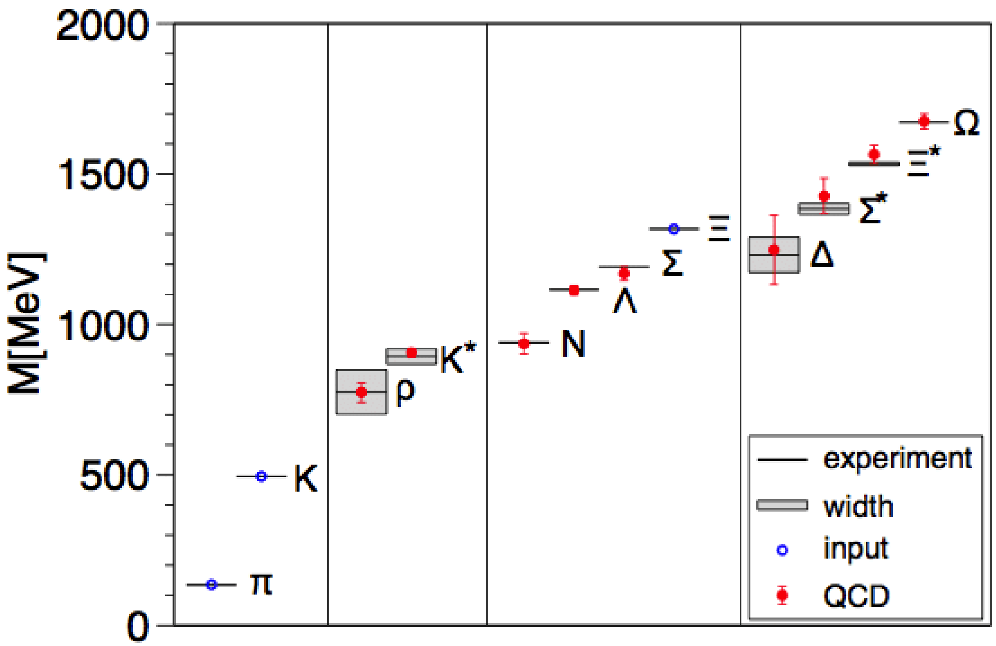
Hadron structure with GPDs (4/5)

Cédric Lorcé

Mass decomposition

Hadron spectroscopy

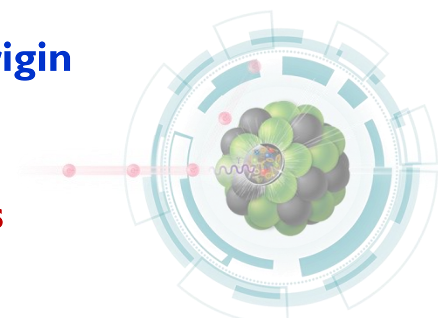
Lattice QCD reproduces very well the light hadron spectrum



[Durr *et al.*, Science 322 (2008) 1224]

… but this does not tell us much about the origin of the hadron masses

One of the goals of the EIC is to provide clues to this fundamental question



What is mass ?

Stereogram → squint

What is mass ?

In relativity, there are essentially *two* equivalent definitions of mass

« Formal » definition $\boxed{p^\mu p_\mu = M^2}$

A global Lorentz-invariant quantity characterizing the physical system

Not additive ! $p^\mu = p^\mu_q + p^\mu_g \quad \Rightarrow \quad M^2 = p_q^2 + p_g^2 + 2\, p_q \cdot p_g$

« Physical » definition $\boxed{p^\mu u_\mu = M}$

Proper inertia (i.e. rest-frame energy) of the system

CM four-velocity $u^\mu = p^\mu / M$

Additive $p^\mu = p^\mu_q + p^\mu_g \quad \Rightarrow \quad M = p_q \cdot u + p_g \cdot u$

$$= \quad p_q^0 \quad + \quad p_g^0$$ **Rest frame**

What is mass ?

Poincaré symmetry tells us that $\langle p|T^{\mu\nu}(0)|p\rangle = 2p^\mu p^\nu$ $\qquad \langle p'|p\rangle = 2p^0(2\pi)^3\delta^{(3)}(\vec{p}\,' - \vec{p})$

« Formal » definition $\langle p|T^\mu{}_\mu(0)|p\rangle = 2M^2$

« Physical » definition $\langle p_{\rm rest}|T^{00}(0)|p_{\rm rest}\rangle = 2M^2$

In the literature one often introduces an *ad hoc* $\frac{1}{2M}$ normalization factor

⚠ **Only expectation values correspond to physical quantities !**

(State normalization is a pure human convention)

➡ **« Formal » definition**

$$\frac{\langle p|\int \mathrm{d}\mathcal{V}\, T^\mu{}_\mu(x)|p\rangle}{\langle p|p\rangle} = M, \qquad \mathrm{d}\mathcal{V} = u^0\,\mathrm{d}^3x$$

Proper volume element

➡ **« Physical » definition**

$$\langle H\rangle\big|_{\rm rest} = \frac{\langle p_{\rm rest}|\int \mathrm{d}^3x\, T^{00}(x)|p_{\rm rest}\rangle}{\langle p_{\rm rest}|p_{\rm rest}\rangle} = M$$

$u^\mu = p^\mu/M$

$$\langle P^\mu\rangle u_\mu = \frac{\langle p|\int \mathrm{d}^3x\, T^{0\mu}(x)|p\rangle}{\langle p|p\rangle}\, u_\mu = M$$

Trace decomposition

The behavior under space-time dilations is determined by

$$j_D^\mu = T^{\mu\nu} x_\nu \qquad \Rightarrow \qquad \partial_\mu j_D^\mu = T^\mu{}_\mu$$

Quark mass and quantum corrections break conformal symmetry

$$T^\mu{}_\mu = \underbrace{\left[\frac{\beta(g)}{2g}\, G^2 + \gamma_m\, \overline{\psi} m \psi\right]}_{\text{Trace anomaly}} + \overline{\psi} m \psi$$

Quark mass matrix

[Crewther, PRL28 (1972) 1421]
[Chanowitz, Ellis, PRD7 (1972) 2490]
[Adler, Collins, Duncan, PRD15 (1977) 1712]
[Collins, Duncan, Joglekar, PRD16 (1977) 438]
[Nielsen, NPB120 (1977) 212]

$$M = \underbrace{\langle \textstyle\int \mathrm{d}\mathcal{V} \left(\frac{\beta(g)}{2g}\, G^2 + \gamma_m\, \overline{\psi} m \psi \right) \rangle}_{\sim 92\%} + \underbrace{\langle \textstyle\int \mathrm{d}\mathcal{V}\, \overline{\psi} m \psi \rangle}_{\sim 8\%}$$

$\mu = 2\,\mathrm{GeV}$

~ 92% (to be measured)

~ 8% (measurement to be improved)

$\langle \overline{\psi} m \psi \rangle$ **Nucleon-meson scattering**

$\langle G^2 \rangle$ **Near-threshold heavy meson production**

Based on this picture, one often concludes that most of the hadron mass comes from gluons !

[Shifman, Vainshtein, Zakharov, PLB78 (1978) 443]
[Donoghue, Golowich, Holstein, CMPPNPC2 (1992) 1]
[Kharzeev, PISPF130 (1996) 105]
[Hatta, Rajan, Tanaka, JHEP12 (2018) 008]

Trace decomposition

The physical interpretation of the « quark » and « gluon » contributions is, however, not so clear …

Rest frame

$$\underbrace{\langle \int d^3x\, T^{\mu}{}_{\mu} \rangle}_{=M} = \underbrace{\langle \int d^3x\, T^{00} \rangle}_{=M} - \sum_i \underbrace{\langle \int d^3x\, T^{ii} \rangle}_{=0}$$

Mechanical equilibrium
(virial theorem)

$T^{\mu\nu} = \sum_a T_a^{\mu\nu}$

$$\underbrace{\langle \int d^3x\, T^{\mu}_{a\mu} \rangle}_{\text{Can be negative !}} = \underbrace{\langle \int d^3x\, T^{00}_{a} \rangle}_{=\langle H_a \rangle} - \sum_i \underbrace{\langle \int d^3x\, T^{ii}_{a} \rangle}_{\neq 0} \qquad a = q, g$$

Partial pressure-volume work

The « gluon » contribution is amplified because the gluon pressure-volume work is negative **(tensile stress)**

[C.L., EPJC78 (2018) 2, 120]

Energy decomposition

Renormalized QCD operators $T^{\mu\nu} = T^{\mu\nu}_q + T^{\mu\nu}_g$

$$T^{\mu\nu}_q = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi$$
$$T^{\mu\nu}_g = -G^{\mu\lambda}G^\nu{}_\lambda + \tfrac{1}{4}\,g^{\mu\nu}\,G^2$$

Rest-frame energy

$$\begin{aligned} M &= \langle H_q\rangle + \langle H_g\rangle \\ &= \underbrace{\langle \textstyle\int \mathrm{d}^3x\,\overline{\psi}\gamma^0 iD^0\psi\rangle}_{[A_q(0)+\bar{C}_q(0)]\,M} + \underbrace{\langle \textstyle\int \mathrm{d}^3x\,\tfrac{1}{2}(\vec{E}^2+\vec{B}^2)\rangle}_{[A_g(0)+\bar{C}_g(0)]\,M} \end{aligned}$$

$$A_a(0) = \langle x\rangle_a$$
$$\bar{C}_a(0) = f_a(\langle x\rangle_q, \langle\overline{\psi}m\psi\rangle)$$

Known but scheme and scale-dependent !

Refinement

$$M = \underbrace{\langle \textstyle\int \mathrm{d}^3x\,\overline{\psi}\vec{\gamma}\cdot i\vec{D}\psi\rangle}_{\substack{\sim 21\%\ (\overline{\text{MS}}) \\ \sim 38\%\ (\text{D2})}} + \underbrace{\langle \textstyle\int \mathrm{d}^3x\,\overline{\psi}m\psi\rangle}_{\substack{\sim 8\%\ (\overline{\text{MS}}) \\ \sim 8\%\ (\text{D2})}} + \underbrace{\langle \textstyle\int \mathrm{d}^3x\,\tfrac{1}{2}(\vec{E}^2+\vec{B}^2)\rangle}_{\substack{\sim 71\%\ (\overline{\text{MS}}) \\ \sim 54\%\ (\text{D2})}}$$

$\mu = 2\,\mathrm{GeV}$

[C.L., EPJC78 (2018) 2, 120]
[Metz, Pasquini, Rodini, PRD102 (2020) 114042]
[C.L., Metz, Pasquini, Rodini, JHEP11 (2021) 121]

Ji's decomposition

Combination of features from *both* trace and energy decompositions

Step 1 $$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \tfrac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$ Twist-2

$$\hat{T}^{\mu\nu} = \tfrac{1}{4}\, g^{\mu\nu} T^{\alpha}{}_{\alpha}$$ Twist-4

Poincaré symmetry ensures that this separation is scheme and scale-independent !

Step 2

$$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g$$

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_m + \hat{T}^{\mu\nu}_{\mathrm{a}}$$

$$\hat{T}^{\mu\nu}_m = \tfrac{1}{4}\, g^{\mu\nu}\, \overline{\psi} m \psi$$

$$\hat{T}^{\mu\nu}_{\mathrm{a}} = \tfrac{1}{4}\, g^{\mu\nu} \left[\frac{\beta(g)}{2g}\, G^2 + \gamma_m\, \overline{\psi} m \psi\right]$$

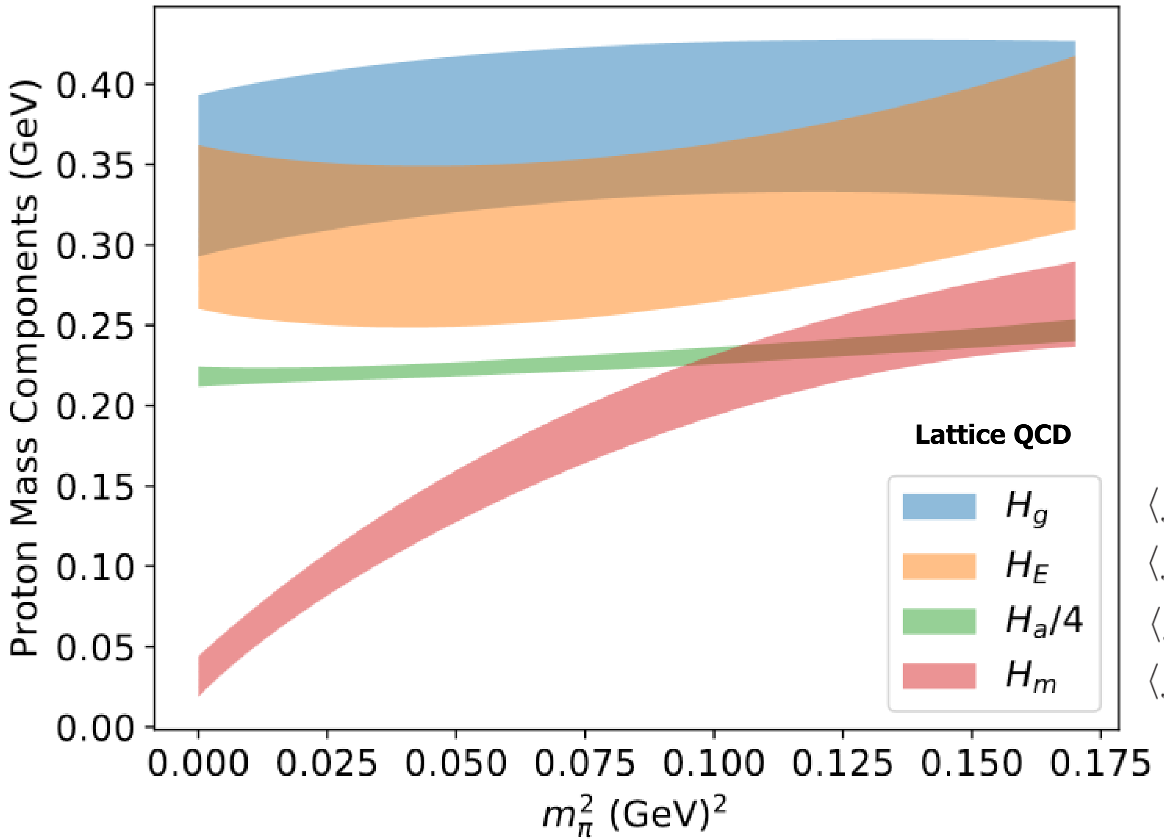
Rest-frame energy

$$M = \langle \textstyle\int \mathrm{d}^3x\, (\bar{T}^{00}_q - \tfrac{3}{4}\, \overline{\psi} m \psi) \rangle + \langle \textstyle\int \mathrm{d}^3x\, \overline{\psi} m \psi \rangle + \langle \textstyle\int \mathrm{d}^3x\, \bar{T}^{00}_g \rangle + \boxed{\langle \textstyle\int \mathrm{d}^3x\, \hat{T}^{00}_{\mathrm{a}} \rangle}$$

« Quantum anomalous energy »

[Ji, PRL74 (1995) 7, 1071]
[Ji, PRD52 (1995) 1, 271]
[Ji, FP16 (2021) 6, 64601]

Ji's decomposition



$\langle \int \mathrm{d}^3x\, \bar{T}_g^{00} \rangle$

$\langle \int \mathrm{d}^3x\, (\bar{T}_q^{00} - \frac{3}{4}\, \overline{\psi} m \psi) \rangle$

$\langle \int \mathrm{d}^3x\, \hat{T}_{\mathrm{a}}^{00} \rangle$ ← **Determined by mass sum rule**

$\langle \int \mathrm{d}^3x\, \overline{\psi} m \psi \rangle$

[Yang *et al.*, PRL121 (2018) 21, 212001]

Ji's decomposition

The physical interpretation of the « quark » and « gluon » contributions is, however, not so clear …

$$T^{00}_a = \underbrace{\bar{T}^{00}_a}_{= \frac{3}{4} T^{00}_a + \boxed{\frac{1}{4}\sum_i T^{ii}_a}} + \underbrace{\hat{T}^{00}_a}_{= \frac{1}{4} T^{00}_a \boxed{- \frac{1}{4}\sum_i T^{ii}_a}} \qquad a = q, g$$

$$\sum_i \langle \textstyle\int \mathrm{d}^3x\, T^{ii}_a \rangle \neq 0$$

Partial pressure-volume work

Also, it is tempting to write the classical relations

$$\bar{T}^{00}_q - \frac{3}{4}\,\overline{\psi} m \psi \overset{?}{=} \overline{\psi}\vec{\gamma}\cdot i\vec{D}\psi$$

$$\bar{T}^{00}_g \overset{?}{=} \frac{1}{2}(\vec{E}^2 + \vec{B}^2)$$

… but there is no scheme (preserving Poincaré symmetry) **where both are simultaneously valid !**

[C.L., EPJC78 (2018) 2, 120]
[Metz, Pasquini, Rodini, PRD102 (2020) 114042]
[C.L., Metz, Pasquini, Rodini, JHEP11 (2021) 121]

Mass decompositions (in D2 scheme $g_{\mu\nu}\langle T_q^{\mu\nu}\rangle = [A_q(0)+4\bar{C}_q(0)]M = \sigma_q$)

Trace decomposition

1 independent input

$$M = \underbrace{\langle \int \mathrm{d}^3x\, \overline{\psi} m \psi\rangle}_{\sigma_q} + \underbrace{\langle \int \mathrm{d}^3x \,(\frac{\beta(g)}{2g}\, G^2 + \gamma_m \overline{\psi} m \psi)\rangle}_{M-\sigma_q}$$

Energy decomposition

2 independent inputs

$$M = \underbrace{\langle \int \mathrm{d}^3x \,(T_q^{00} - \overline{\psi} m \psi)\rangle}_{[A_q(0)+\bar{C}_q(0)]\,M-\sigma_q} + \underbrace{\langle \int \mathrm{d}^3x\, \overline{\psi} m \psi\rangle}_{\sigma_q} + \underbrace{\langle \int \mathrm{d}^3x\, T_g^{00}\rangle}_{[A_g(0)+\bar{C}_g(0)]\,M}$$

$$A_q(0) + A_g(0) = 1$$
$$\bar{C}_q(0) + \bar{C}_g(0) = 0$$

Ji's decomposition

2 independent inputs

$$M = \underbrace{\langle \int \mathrm{d}^3x \,(\bar{T}_q^{00} - \tfrac{3}{4}\, \overline{\psi} m \psi)\rangle}_{\frac{3}{4}[A_q(0)\,M-\sigma_q]} + \underbrace{\langle \int \mathrm{d}^3x\, \overline{\psi} m \psi\rangle}_{\sigma_q} + \underbrace{\langle \int \mathrm{d}^3x\, \bar{T}_g^{00}\rangle}_{\frac{3}{4}A_g(0)\,M} + \underbrace{\langle \int \mathrm{d}^3x\, \tfrac{1}{4}(\frac{\beta(g)}{2g}\, G^2 + \gamma_m \overline{\psi} m \psi)\rangle}_{\frac{1}{4}(M-\sigma_q)}$$

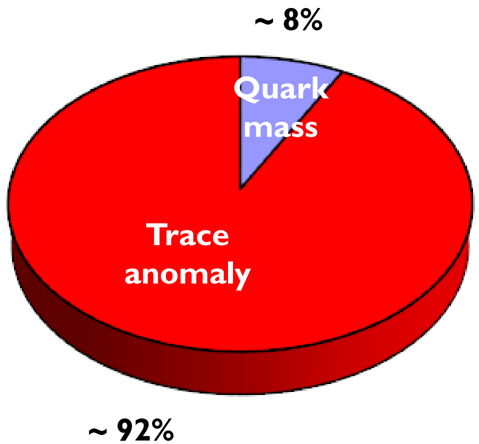
Mechanical equilibrium
(virial theorem)

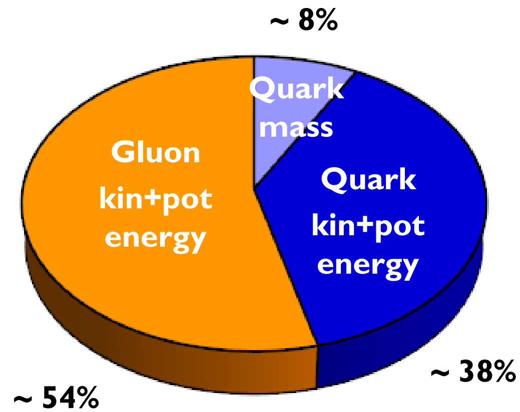
$$\langle \int \mathrm{d}^3x\, T^{ii}\rangle = 0 \;\Rightarrow\; \boxed{\langle \int \mathrm{d}^3x \,(\bar{T}_q^{00} - \tfrac{3}{4}\, \overline{\psi} m \psi)\rangle + \langle \int \mathrm{d}^3x\, \bar{T}_g^{00}\rangle = 3\,\langle \int \mathrm{d}^3x\, \hat{T}_{\mathrm{a}}^{00}\rangle}$$

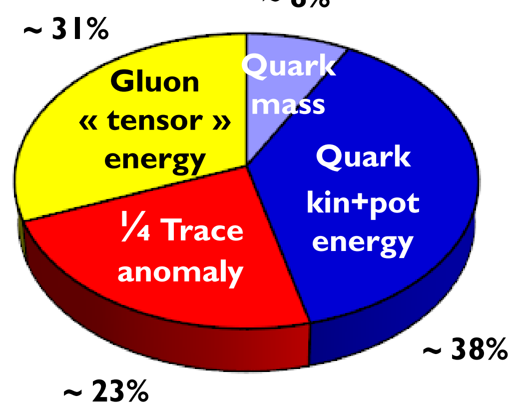
Mass decompositions (in D2 scheme $g_{\mu\nu}\langle T_q^{\mu\nu}\rangle = [A_q(0) + 4\bar{C}_q(0)]M = \sigma_q$)

Trace decomposition | **Energy decomposition** | **Ji's decomposition**

$\mu = 2\,\mathrm{GeV}$







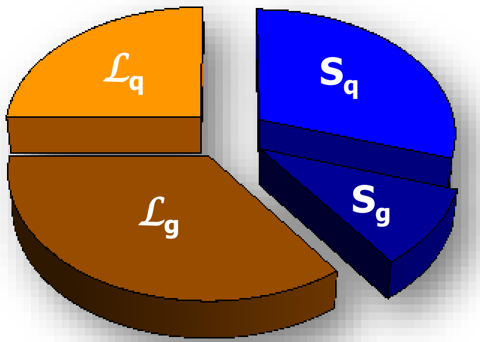
	Trace decomposition	Energy decomposition	Ji's decomposition
Scale dependence	No	Yes	Yes
Relation to mass	Amplitude level	Operator level	Operator level
Virial theorem	Dependent	Independent	Dependent

Spin decomposition

Spin sum rules (1990-2008)

Canonical

$\vec{p} = \frac{\partial L}{\partial \vec{v}}$



$$\vec{S}_q = \int \mathrm{d}^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{\mathcal{L}}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_g = \int \mathrm{d}^3r\, \vec{E}^a \times \vec{A}^a$$

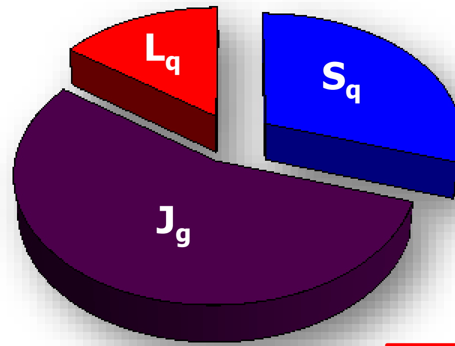
$$\vec{\mathcal{L}}_g = \int \mathrm{d}^3r\, E^{ai}(\vec{r} \times \vec{\nabla})A^{ai}$$

Gauge fixed ! $\boxed{A^+ = 0}$

[Jaffe, Manohar, NPB337 (1990) 509]

Kinetic

$\vec{\pi} = m\vec{v}$
$= \vec{p} + g\vec{A}$



$\boxed{\vec{D} = -\vec{\nabla} - ig\vec{A}}$

$$\vec{S}_q = \int \mathrm{d}^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{L}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{J}_g = \int \mathrm{d}^3r\, \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

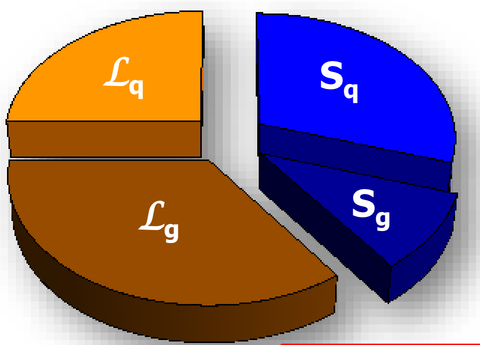
« Incomplete » ΔG **?**

[Ji, PRL78 (1997) 610]

Spin sum rules (2008-now)

$$\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}$$

Canonical



$$\boxed{\vec{D}_{\text{pure}} = -\vec{\nabla} - ig\vec{A}_{\text{pure}}}$$

$$\vec{S}_q = \int d^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{\mathcal{L}}_q = \int d^3r\, \psi^\dagger \vec{r} \times (i\vec{D}_{\text{pure}})\psi$$

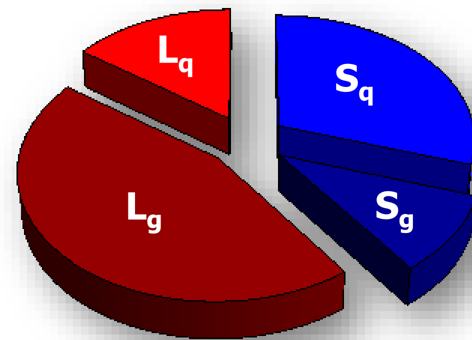
$$\vec{S}_g = \int d^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{\mathcal{L}}_g = -\int d^3r\, E^{ai}(\vec{r} \times \vec{\mathcal{D}}^{ab}_{\text{pure}})A^{bi}_{\text{phys}}$$

Gauge invariant !

[Chen, Lu, Sun, Wang, Goldstein, PRL100 (2008) 232002]

Kinetic



$$\boxed{\vec{D} = -\vec{\nabla} - ig\vec{A}}$$

$$\vec{S}_q = \int d^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{L}_q = \int d^3r\, \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_g = \int d^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{L}_g = \vec{\mathcal{L}}_g - \int d^3r\, \rho^a \vec{r} \times \vec{A}^a_{\text{phys}}$$

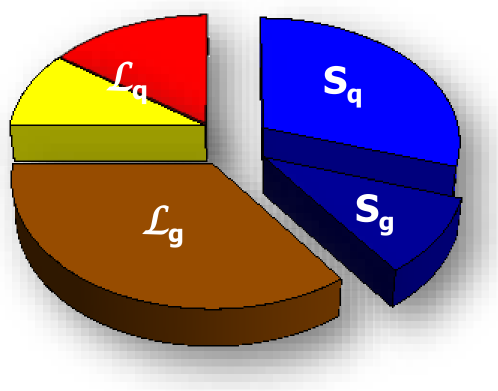
« Complete »

[Wakamatsu, PRD81 (2010) 114010]

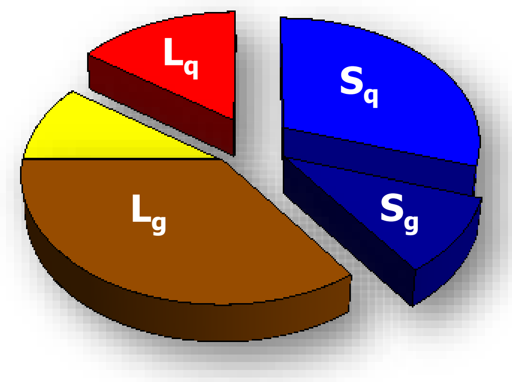
Spin sum rules (2008-now)

$$\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}$$

Canonical



Kinetic



$$\vec{S}_q = \int \mathrm{d}^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{L}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_g = \int \mathrm{d}^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{\mathcal{L}}_g = -\int \mathrm{d}^3r\, E^{ai}(\vec{r} \times \vec{\mathcal{D}}^{ab}_{\text{pure}}) A^{ai}_{\text{phys}}$$

« Potential »
angular momentum

$$\vec{L}_{\text{pot}} = -\int \mathrm{d}^3r\, \rho^a \vec{r} \times \vec{A}^a_{\text{phys}}$$

[Wakamatsu, PRD81 (2010) 114010]
[C.L., Leader, PR541 (2014) 163]

Ambiguity

$$\boxed{A^\mu = A^\mu_{\text{pure}} + A^\mu_{\text{phys}}}$$

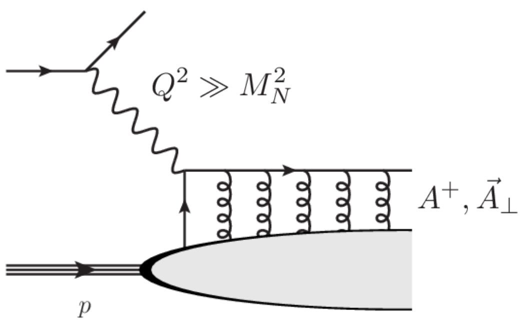
Pure gauge field $A^\mu_{\text{pure}} \equiv U \frac{i}{g} \partial^\mu U^{-1} \quad \Leftrightarrow \quad F^{\mu\nu}_{\text{pure}} = 0$

$$F^{\mu\nu} = \mathcal{D}^\mu_{\text{pure}} A^\nu_{\text{phys}} - \mathcal{D}^\nu_{\text{pure}} A^\mu_{\text{phys}} - ig[A^\mu_{\text{phys}}, A^\nu_{\text{phys}}]$$

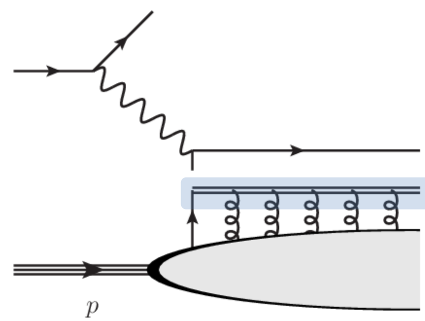
The split is not unique and reflects the original gauge symmetry

In general, A^μ_{pure} and A^μ_{phys} are non-local functionals of A^μ

Factorization in high-energy scattering



$\approx$



$+$ **higher twists**

$$\mathcal{W} = \mathcal{P}\left[\exp\left(ig \int \mathrm{d}y^- A^+\right)\right]$$

Non-local

$$A^{\text{pure}}_\mu = \mathcal{W} \frac{i}{g} \partial_\mu \mathcal{W}^{-1}$$

$$A^+ = A^+_{\text{pure}} \quad \Rightarrow \quad A^+_{\text{phys}} = 0$$

[Hatta, PLB708 (2012) 186]
[C.L., PLB719 (2013) 185]
[C.L., PRD187 (2013) 034031]

Generalized angular momentum

Poincaré symmetry implies that the following current is conserved

$$J^{\mu\alpha\beta} = \underbrace{x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}}_{\text{Orbital}} + \underbrace{S^{\mu\alpha\beta}}_{\text{Intrinsic}}$$

$$\left.\begin{aligned} \partial_\mu J^{\mu\alpha\beta} &= 0 \\ \partial_\mu T^{\mu\nu} &= 0 \end{aligned}\right\} \Rightarrow \boxed{T^{[\alpha\beta]} = -\partial_\mu S^{\mu\alpha\beta}}$$

$$x^{\{\mu}y^{\nu\}} = x^\mu y^\nu + x^\nu y^\mu$$
$$x^{[\mu}y^{\nu]} = x^\mu y^\nu - x^\nu y^\mu$$

$$\Gamma^{\mu\nu}_a(P,\Delta) = \frac{P^\mu P^\nu}{M}A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a(t) + Mg^{\mu\nu}\bar{C}_a(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M}J_a(t) - \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{2M}S_a(t)$$

$$\Rightarrow \boxed{\begin{aligned} S_q(t) &= \tfrac{1}{2}\,G_A^q(t) \\ S_g(t) &= 0 \end{aligned}}$$

[Shore, White, NPB581 (2000) 409]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]
[C.L., Leader, PR541 (2014) 163]

Forward matrix elements of angular momentum are ill-defined

$x^0 = 0$

$$\langle p|\int \mathrm{d}^3x\,x^i O(x)|p\rangle = \langle p|O(0)|p\rangle \underbrace{\int \mathrm{d}^3x\,x^i}_{\text{Ambiguous !}}$$

Amplitude for compound operators

Standard prescription

$$\langle \int d^3x\, x^i O(x)\rangle \equiv \lim_{\Delta\to 0} \frac{\langle P+\frac{\Delta}{2}|\int d^3x\, x^i O(x)|P-\frac{\Delta}{2}\rangle}{\langle p|p\rangle}$$

$$= \underbrace{\frac{\langle p|O(0)|p\rangle}{\langle p|p\rangle}(2\pi)^3 i\nabla^i\delta^{(3)}(\vec{0})}_{\text{Contribution from the « center » of the wave packet}} + \underbrace{\frac{1}{2p^0}\left[-i\nabla^i_\Delta\langle P+\frac{\Delta}{2}|O(0)|P-\frac{\Delta}{2}\rangle\right]_{\Delta=0}}_{\text{Internal contribution}}$$

Contribution from the « center » of the wave packet

Internal contribution

[Jaffe, Manohar, NPB337 (1990) 509]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

Phase-space approach

$$\langle \int d^3x\, x^i O(x)\rangle_{\vec{R},\vec{P}} = R^i\frac{\langle p|O(0)|p\rangle}{2p^0} + \underbrace{\frac{1}{2p^0}\left[-i\nabla^i_\Delta\langle P+\frac{\Delta}{2}|O(0)|P-\frac{\Delta}{2}\rangle\right]_{\Delta=0}}_{=\langle\int d^3r\, r^i O(r)\rangle_{\vec{0},\vec{P}}}$$

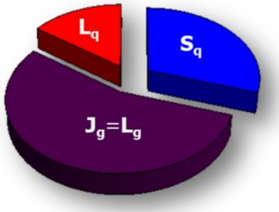
Internal contribution

Remark: $\langle O\rangle_{\text{standard}} = \int \frac{d^3R}{(2\pi)^3\,\delta^{(3)}(\vec{0})}\langle O\rangle_{\vec{R},\vec{P}}$ explains the origin of the delta contribution in standard approach

[C.L., EPJC78 (2018) 9, 785]
[C.L., EPJC81 (2021) 5, 431]

Angular momentum conservation *(local kinetic operators)*

Longitudinally polarized target $\langle J_z\rangle = \langle L_z^q\rangle + \langle S_z^q\rangle + \langle L_z^g\rangle$



For a wave-packet centered at the origin, one finds

$$\langle L_z^q\rangle = \langle \int \mathrm{d}^3x\ (x^1 T_q^{02} - x^2 T_q^{01})\rangle = J_q(0) - S_q(0)$$

$$\langle S_z^q\rangle = \tfrac{1}{2}\,\Delta\Sigma = \tfrac{1}{2}\,G_A^q(0) = S_q(0)$$

$$\langle L_z^g\rangle = \langle \int \mathrm{d}^3x\ (x^1 T_g^{02} - x^2 T_g^{01})\rangle = J_g(0)$$

$S_g(t) = 0$

Angular momentum (or spin) sum rule

$$\boxed{\sum_a J_a(0) = \frac{1}{2}}$$

[Ji, PRL78 (1997) 610]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

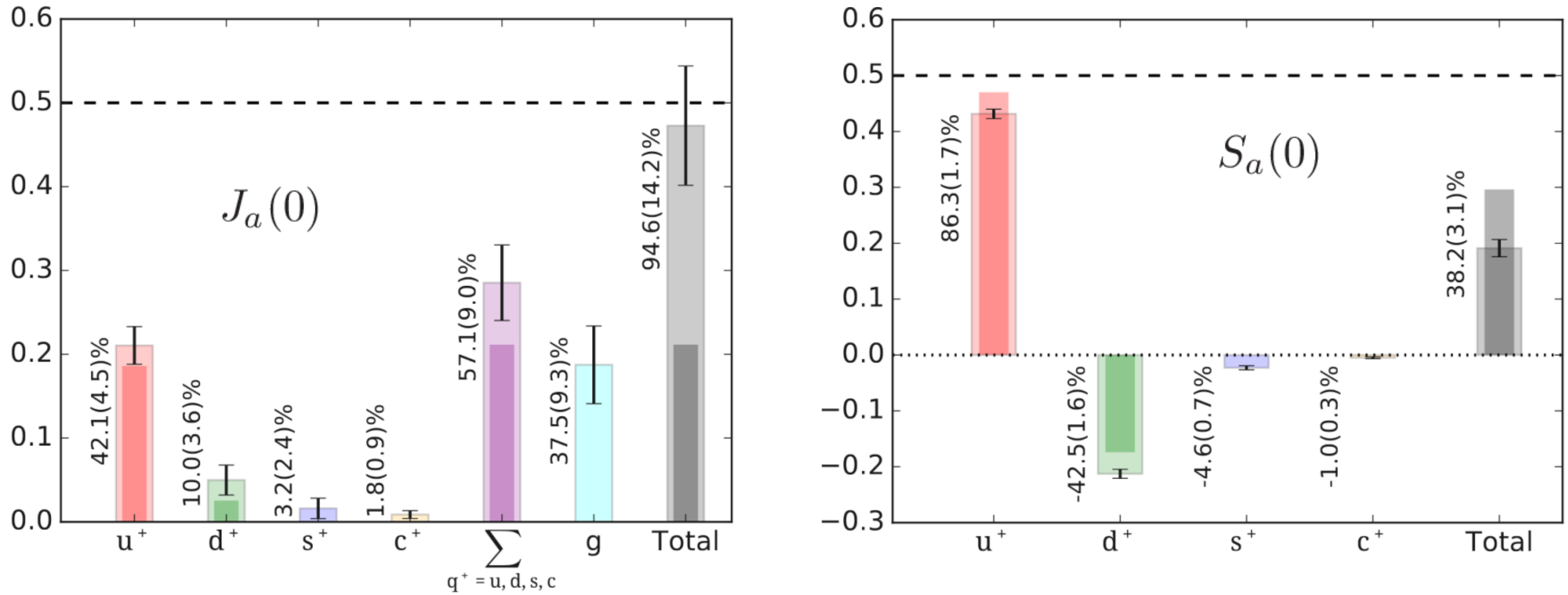
The same sum rule is obtained with a transversely polarized target

(but more complicated analysis due to non-commutativity of boosts)

[C.L., EPJC81 (2021) 5, 431]

Angular momentum conservation *(local kinetic operators)*

Lattice QCD has made a lot of progress recently in computing the nucleon spin contributions



[Alexandrou *et al.*, PRD101 (2020) 094513]

Anomalous gravitomagnetic moment

Mellin moments $\xi = 0$

$$\int \mathrm{d}x\, H_q(x,0,t) = F_1^q(t) \qquad \int \mathrm{d}x\, x\, H_a(x,0,t) = A_a(t)$$

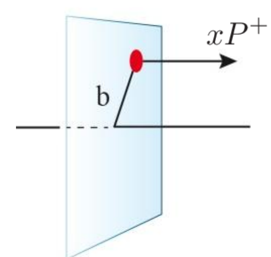
$$\int \mathrm{d}x\, E_q(x,0,t) = F_2^q(t) \qquad \int \mathrm{d}x\, x\, E_a(x,0,t) = 2J_a(t) - A_a(t) \equiv B_a(t)$$

Anomalous magnetic moment $F_2^q(0) = \kappa_q$

Anomalous gravitomagnetic moment $B_a(0) = \kappa_a^G$

Sum rule

$$\left.\begin{aligned}\sum_a A_a(0) &= 1\\ \sum_a J_a(0) &= \tfrac{1}{2}\end{aligned}\right\} \Rightarrow \boxed{\sum_a B_a(0) = 0}$$



$$\vec{R}_\perp = \frac{1}{P^+}\int \mathrm{d}x^-\, \mathrm{d}^2x_\perp\, \vec{x}_\perp\, T^{++}(x)$$

Equivalence principle for spinning targets

$\Rightarrow$ **No intrinsic energy dipole moment !**

[Ji, PRL78 (1997) 610]
[Teryaev, hep-ph/9904376]
[Brodsky, Hwang, Ma, Schmidt, NPB593 (2001) 311]
[Burkardt, IJMPA18 (2003) 2, 173]
[Lowdon, Chiu, Brodsky, PLB774 (2017) 1]
[C.L., EPJC78 (2018) 9, 785]
[Cotogno, C.L., Lowdon, PRD100 (2019) 045003]

Generalizations

Distribution in momentum space

$$\langle J_z^a \rangle = \int \mathrm{d}x\, \underbrace{\frac{x}{2}\left[H_a(x,0,0) + E_a(x,0,0)\right]}_{\equiv J_a(x) \neq \langle J_z^a\rangle(x)}$$

[Hoodbhoy, Ji, Lu, PRD 59 (1999) 014013]
[Hägler, Mukherjee, Schäfer, PLB582 (2004) 55]
[Ji, Xiong, Yuan, PRL109 (2012) 152005]
[Ji, Xiong, Yuan, PLB717 (2012) 214]
[Hatta, Yoshida, JHEP02 (2012) 003]
[Ji, Xiong, Yuan, PRD88 (2013) 1, 014041]
[C.L., PLB719 (2013) 185]
[Liu, C.L., EPJA52 (2016) 6, 160]

$[D^\mu, \mathcal{W}] \neq 0$ → **Non-local version of covariant derivative is ambiguous and involves higher-twist distributions !**

Distribution in position space

$$\langle J_z^a \rangle = J_a(0) = \int \mathrm{d}^3r\, \underbrace{\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, J_a(t)}_{\equiv \mathcal{J}_a(\vec{r}) \neq \langle J_z^a\rangle(\vec{r})}$$

[Polyakov, PLB555 (2003) 57]
[Goeke *et al.*, PRD75 (2007) 094021]
[Adhikari, Burkardt, NPBPS251 (2014) 105]
[Leader, C.L., PR541 (2014) 3, 163]
[Adhikari, Burkardt, PRD94 (2016) 11, 114021]
[Liu, C.L., EPJA52 (2016) 6, 160]
[C.L., Mantovani, Pasquini, PLB776 (2018) 38]

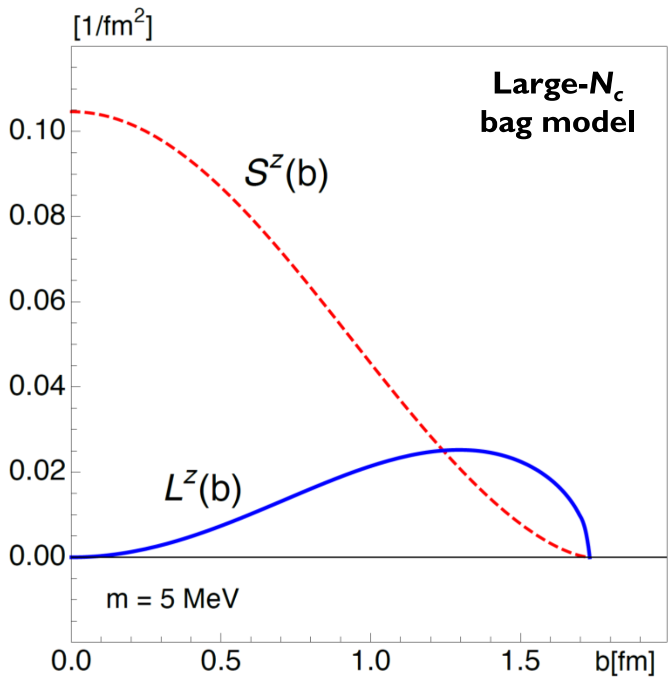
$\vec{\Delta}_\perp^2 \dfrac{\mathrm{d}J_a(t)}{\mathrm{d}t}$ **does not contribute to the integral but affects the spatial distribution !**

Angular momentum distributions

Orbital *vs* intrinsic

$$L^i(\vec{r}) = \epsilon^{ijk} r^j \langle T^{0k}\rangle_{\vec{0},\vec{0}}(\vec{r})$$

$$S^i(\vec{r}) = \tfrac{1}{2}\langle\overline{\psi}\gamma^i\gamma_5\psi\rangle_{\vec{0},\vec{0}}(\vec{r})$$

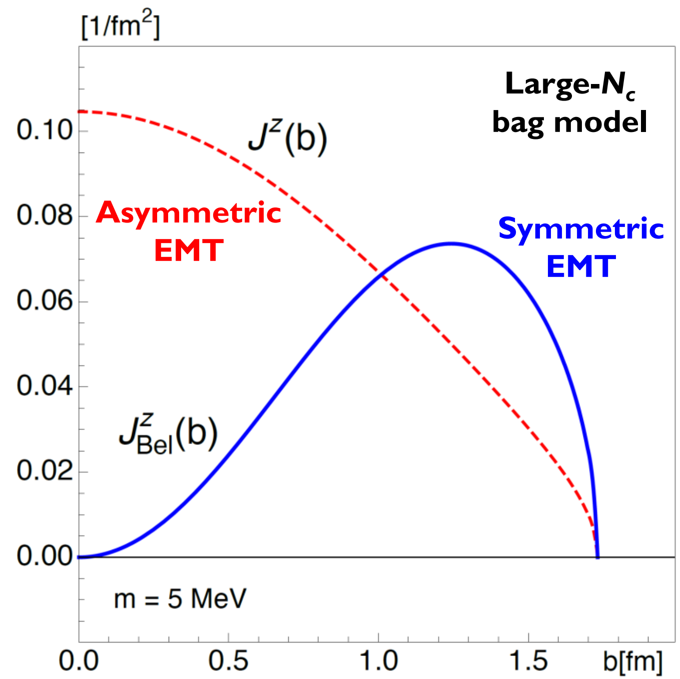


Impact parameter $b = \sqrt{x^2+y^2} = \sqrt{r^2-z^2}$

Kinetic *vs* Belinfante

$$J^i(\vec{r}) = L^i(\vec{r}) + S^i(\vec{r})$$

$$J^i_{\rm Bel}(\vec{r}) = \epsilon^{ijk} r^j \langle\tfrac{1}{2}(T^{0k}+T^{k0})\rangle_{\vec{0},\vec{0}}(\vec{r})$$



[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
[C.L., Schweitzer, Tezgin, PRD106 (2022) 1, 014012]