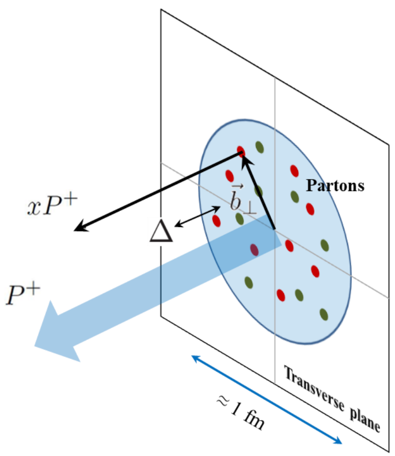
CNF lecture series, July13-17



Hadron structure with GPDs (5/5)

Cédric Lorcé

Mechanical properties

Let us recap

$$\langle p', s' | T_a^{\mu\nu}(0) | p, s \rangle = \overline{u}(p', s') \Gamma_a^{\mu\nu}(P, \Delta) u(p, s)$$

$$\Gamma_a^{\mu\nu}(P,\Delta) = \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu} \bar{C}_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M} J_a(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{2M} S_a(t)$$

$A_a(0) \leftrightarrow$ **Momentum**

$\bar{C}_a(0) \leftrightarrow$ **Pressure**

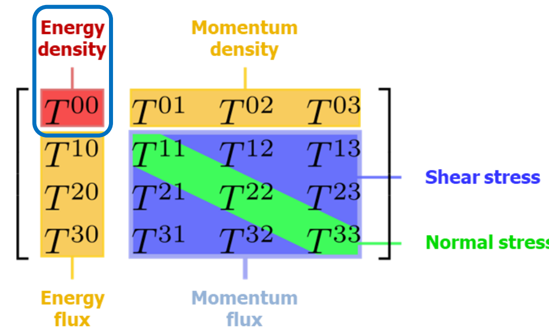
$J_a(0) \leftrightarrow$ **Total angular momentum**

$S_q(0) \leftrightarrow$ **Intrinsic angular momentum**

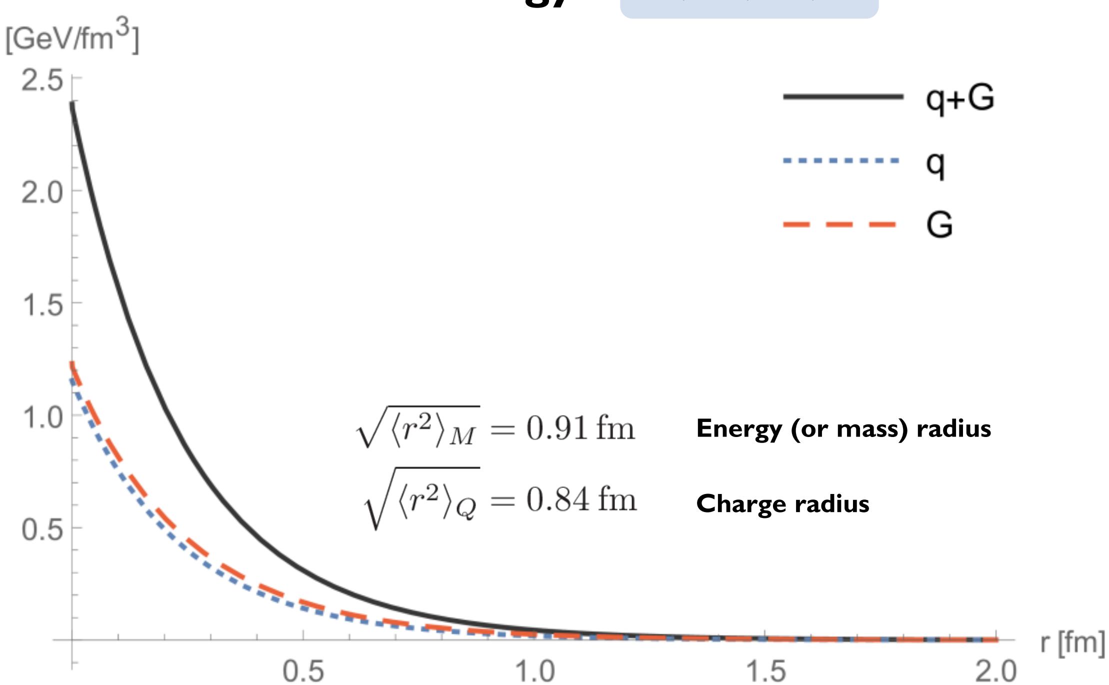
$C_a(0) \leftrightarrow$ **?**

Rest frame energy (or mass) distribution

$$\langle T^{00} \rangle (\vec{r})$$



Energy $A_a, C_a, \bar{C}_a, J_a$

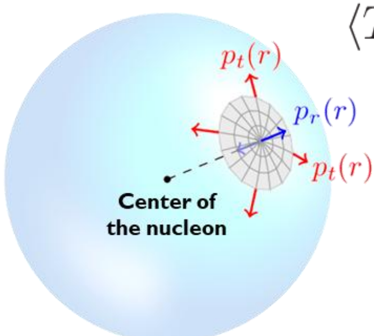


Multipole model for the gravitational form factors

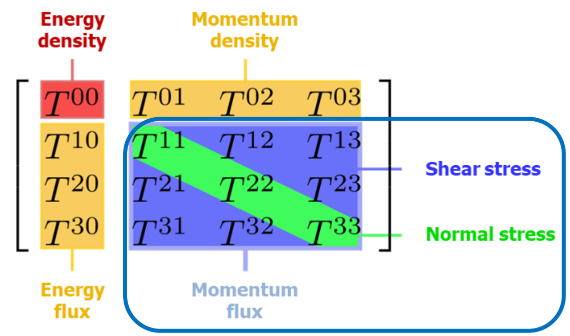
$$F(t) = \frac{F(0)}{(1 + t/\Lambda^2)^n}$$

[Polyakov, PLB555 (2003) 57]
[Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025]
[C.L., Moutarde, Trawinski, EPJC79 (2019) 1, 89]

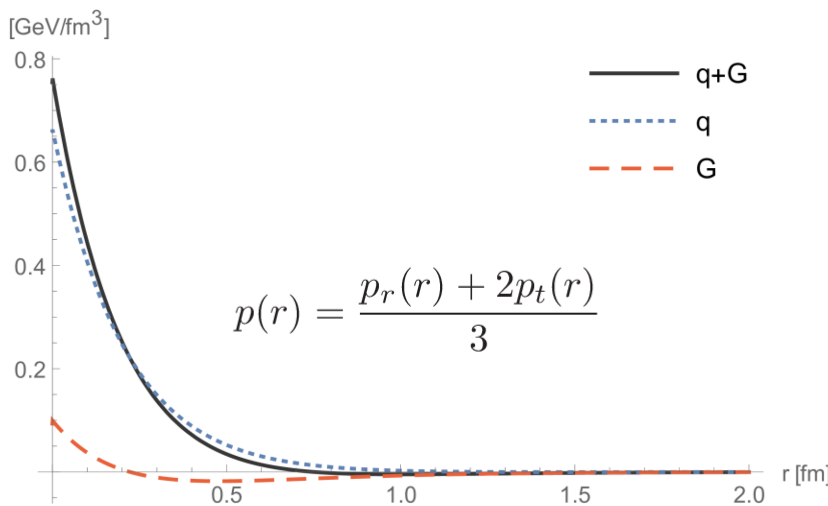
Rest frame pressure distributions



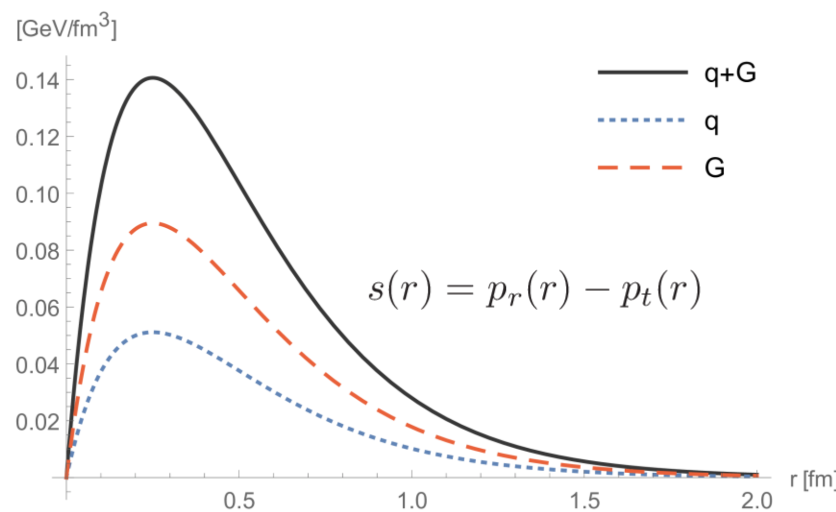
$$\langle T^{ij}\rangle(\vec{r}) = \delta^{ij}\, p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\,\delta^{ij}\right) s(r)$$



Isotropic pressure $C_a, \bar{C}_a$



Pressure anisotropy C_a



[Polyakov, PLB555 (2003) 57]
[Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025]
[C.L., Moutarde, Trawinski, EPJC79 (2019) 1, 89]

Some figures

	Density (kg/m^3)	Pressure (Pa or N/m^2)
Atmosphere at sea level	≈ 1.2	$\approx 10^5$
Center of sun	$\approx 1.6 \times 10^5$	$\approx 2.5 \times 10^{16}$
Center of neutron star	$\approx 8 \times 10^{17}$	$\approx 10^{35}$
Atomic nucleus	$\approx 2.3 \times 10^{17}$	$\approx 3.5 \times 10^{33}$
Center of nucleon	$\approx 3 \times 10^{18}$	$\approx 3.7 \times 10^{34}$
PhD student (one month before graduation)	$\approx 10^3$	$> 10^{42}$

Mechanical equilibrium

$$\nabla^i \langle T^{ij} \rangle (\vec{r}) = 0$$

Young-Laplace equation

$$\frac{\mathrm{d}p_r(r)}{\mathrm{d}r} = -\frac{2s(r)}{r}$$

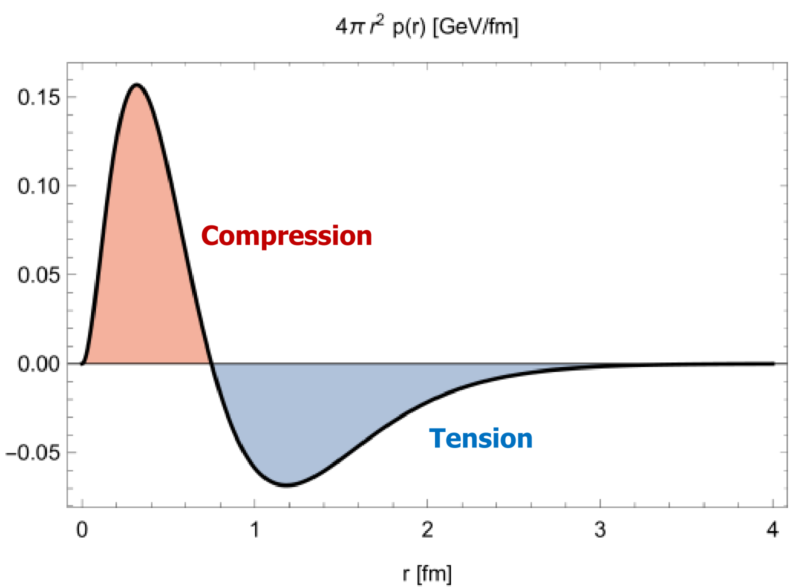
Radial pressure-gradient force density

Capillary force density

von Laue relation

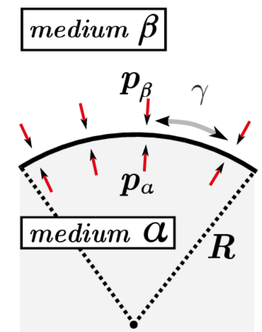
$$\boxed{\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0}$$

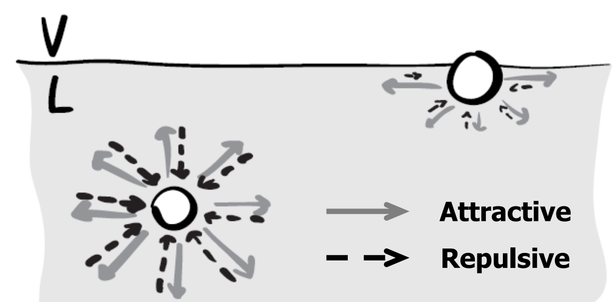
[Laue, AP340 (1911) 8, 524]



Surface tension

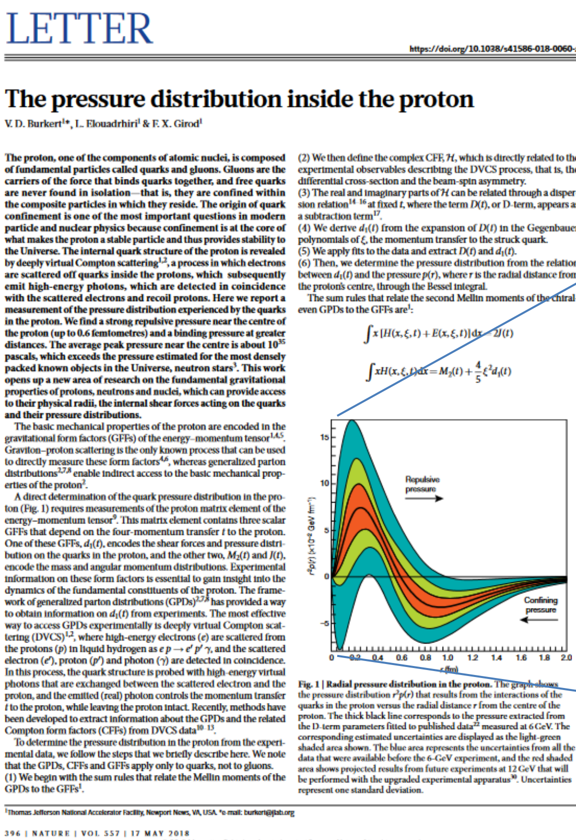
$$\gamma = \int \mathrm{d}r\, s(r)$$





[Bakker, *Kapillarität und Oberflächenspannung* (1928)]
[Kirkwood, Buff, JCP17 (1949) 338]
[Marchand, Weijs, Snoeijer, Andreotti, AJP79 (2011) 999]

First experimental extractions



LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1*], L. Elouadrhiri[1] & F. X. Girod[1]

The proton, one of the components of atomic nuclei, is composed of fundamental particles called quarks and gluons. Gluons are the carriers of the force that binds quarks together, and free quarks are never found in isolation—that is, they are confined within the composite particles in which they reside. The origin of quark confinement is one of the most important questions in modern particle and nuclear physics because confinement is at the core of what makes the proton a stable particle and thus provides stability to the Universe. The internal quark structure of the proton is revealed by deeply virtual Compton scattering[1,2], a process in which electrons are scattered off quarks inside the protons, which subsequently emit high-energy photons, which are detected in coincidence with the scattered electrons and recoil protons. Here we report a measurement of the pressure distribution experienced by the quarks in the proton. We find a strong repulsive pressure near the centre of the proton (up to 0.6 femtometres) and a binding pressure at greater distances. The average peak pressure near the centre is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars[3]. This work opens up a new area of research on the fundamental gravitational properties of protons, neutrons and nuclei, which can provide access to their physical radii, the internal shear forces acting on the quarks and their pressure distributions.

The basic mechanical properties of the proton are encoded in the gravitational form factors (GFFs) of the energy–momentum tensor[1,4,5]. Graviton–proton scattering is the only known process that can be used to directly measure these form factors[4,6], whereas generalized parton distributions[2,7,8] enable indirect access to the basic mechanical properties of the proton[2].

A direct determination of the quark pressure distribution in the proton (Fig. 1) requires measurements of the proton matrix element of the energy–momentum tensor[9]. This matrix element contains three scalar GFFs that depend on the four-momentum transfer t to the proton. One of these GFFs, $d_1(t)$, encodes the shear forces and pressure distribution on the quarks in the proton, and the other two, $M_2(t)$ and $J(t)$, encode the mass and angular momentum distributions. Experimental information on these form factors is essential to gain insight into the dynamics of the fundamental constituents of the proton. The framework of generalized parton distributions (GPDs)[2,7,8] has provided a way to obtain information on $d_1(t)$ from experiments. The most effective way to access GPDs experimentally is deeply virtual Compton scattering (DVCS)[1,2], where high-energy electrons (e) are scattered from the protons (p) in liquid hydrogen as $e\,p \rightarrow e'\,p'\,\gamma$, and the scattered electron (e'), proton (p') and photon (γ) are detected in coincidence. In this process, the quark structure is probed with high-energy virtual photons that are exchanged between the scattered electron and the proton, and the emitted (real) photon controls the momentum transfer t to the proton, while leaving the proton intact. Recently, methods have been developed to extract information about the GPDs and the related Compton form factors (CFFs) from DVCS data[10–13].

To determine the pressure distribution in the proton from the experimental data, we follow the steps that we briefly describe here. We note that the GPDs, CFFs and GFFs apply only to quarks, not to gluons.

(1) We begin with the sum rules that relate the Mellin moments of the GPDs to the GFFs[1].

(2) We then define the complex CFF, $\mathcal{H}$, which is directly related to the experimental observables describing the DVCS process, that is, the differential cross-section and the beam-spin asymmetry.

(3) The real and imaginary parts of $\mathcal{H}$ can be related through a dispersion relation[14–16] at fixed t, where the term $D(t)$, or D-term, appears as a subtraction term[17].

(4) We derive $d_1(t)$ from the expansion of $D(t)$ in the Gegenbauer polynomials of ξ, the momentum transfer to the struck quark.

(5) We apply fits to the data and extract $D(t)$ and $d_1(t)$.

(6) Then, we determine the pressure distribution from the relation between $d_1(t)$ and the pressure $p(r)$, where r is the radial distance from the proton's centre, through the Bessel integral.

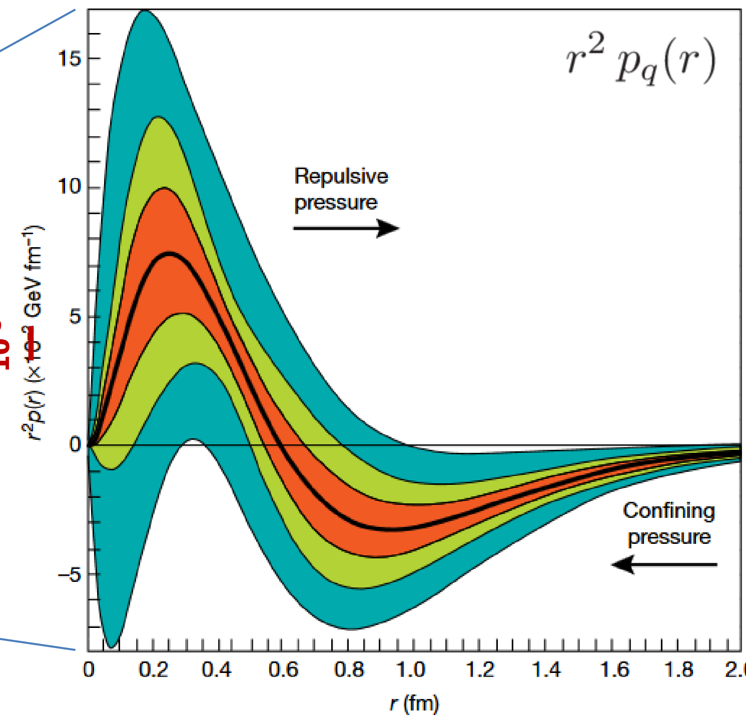
The sum rules that relate the second Mellin moments of the chiral-even GPDs to the GFFs are[1]:

$$\int x\,[H(x,\xi,t)+E(x,\xi,t)]\,\mathrm{d}x = 2J(t)$$

$$\int xH(x,\xi,t)\,\mathrm{d}x = M_2(t) + \frac{4}{5}\xi^2 d_1(t)$$

Fig. 1 | Radial pressure distribution in the proton. The graph shows the pressure distribution $r^2p(r)$ that results from the interactions of the quarks in the proton versus the radial distance r from the centre of the proton. The thick black line corresponds to the pressure extracted from the D-term parameters fitted to published data[22] measured at 6 GeV. The corresponding estimated uncertainties are displayed as the light-green shaded area shown. The blue area represents the uncertainties from all the data that were available before the 6-GeV experiment, and the red shaded area shows projected results from future experiments at 12 GeV that will be performed with the upgraded experimental apparatus[30]. Uncertainties represent one standard deviation.

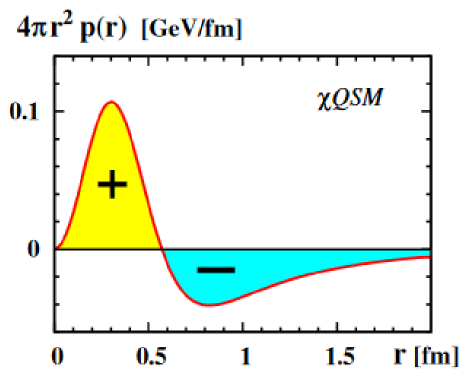
[1]Thomas Jefferson National Accelerator Facility, Newport News, VA, USA. *e-mail: burkert@jlab.org

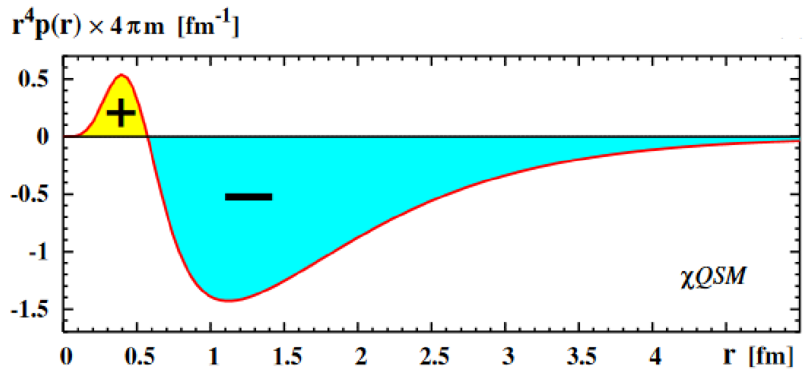


Contribution from $\bar{C}_q(t)$ still missing !

[Burkert, Elouadrhiri, Girod, Nature557 (2018) 7705, 396]
[Kumericki, Nature570 (2019) 7759, E1]
[Dutrieux, C.L., Moutarde, Sznajder, Trawinski, EPJC81 (2021) 4, 300]

D-term and stability





$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

[Laue, AP340 (1911) 8, 524]

$$M \int_0^\infty \mathrm{d}r\, r^4\, p(r) = 4C(0) = D(0)$$

***Druck* =**
« pressure »

[Polyakov, Weiss, PRD60 (1999) 114017]
[Goeke *et al.*, PRD75 (2007) 094021]

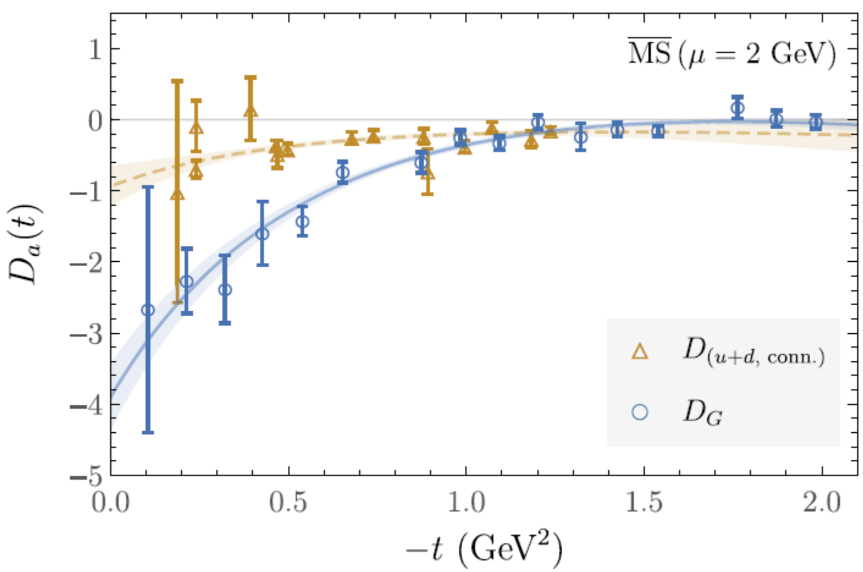
Equilibrium

Stability $\overset{?}{\Leftrightarrow}$ $D(0) < 0$

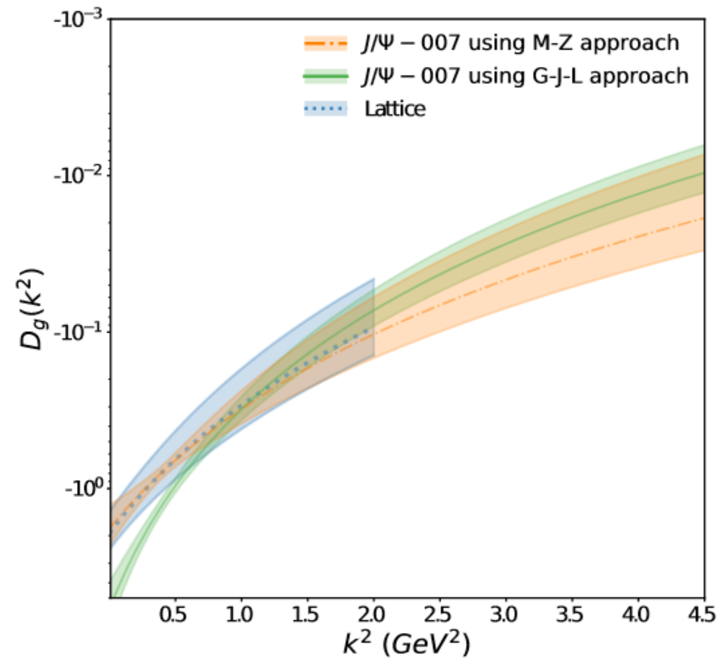
[Perevalova, Polyakov, Schweitzer, PRD94 (2016) 5, 054024]
[Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025]
[Varma, Schweitzer, PRD102 (2020) 1, 014047]

D-term and stability

This conjecture is supported by both Lattice QCD and experiments



[Hägler *et al.*, PRD77 (2008) 094502]
[Shanahan, Detmold, PRD99 (2019) 4, 014511]
[Shanahan, Detmold, PRL122 (2019) 072003]

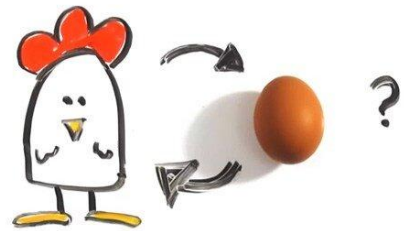


[Duran *et al.*, Nature615 (2023) 7954, 813]
[Meziani, PoS SPIN2023 (2024) 168]

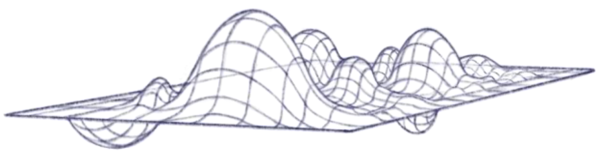
… but does not apply to long-range forces, like Coulomb forces

[Varma, Schweitzer, PRD102 (2020) 1, 014047]
[Ji, Yang, NPB1024 (2026) 117342]
[Meija, Schweitzer, PRD113 (2026) 11, 116008

Validity of the mechanical interpretation



Field picture

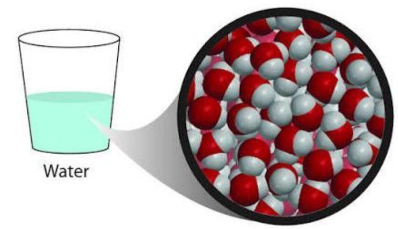


Fields are the fundamental degrees of freedom

- **Continuum mechanics concepts apply as is at microscopic level**
- **Particles emerge as quantized field excitations**

[Polyakov, PLB555 (2003) 57]
[Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025]
[C.L., Schweitzer, APPB56 (2025) 3, 3A-17]
[C.L., Freese, *in preparation* (2026)]

Particle picture



Point particles are fundamental and interact via fields

- **Continuum mechanics description valid only at « macroscopic » level**
- **No « pressure », just momentum flow**

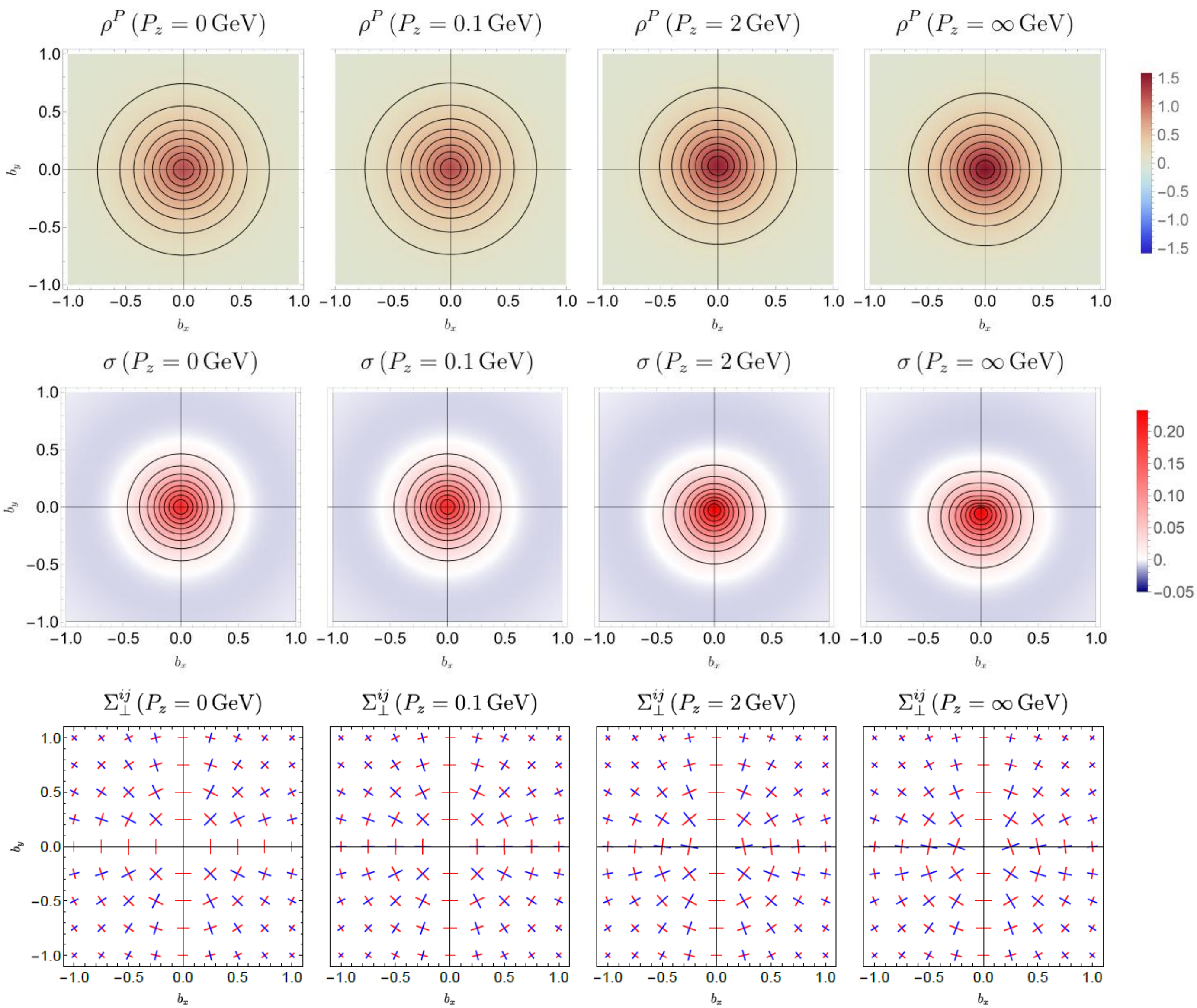
[Ji, Liu, PRD106 (2022) 3, 034028]
[Ji, Yang, R9 (2026) 1155]
[Ji, Yang, NPB1024 (2026) 117342]

Frame dependence (longitudinal polarization)

Energy distribution

2D isotropic pressure distribution

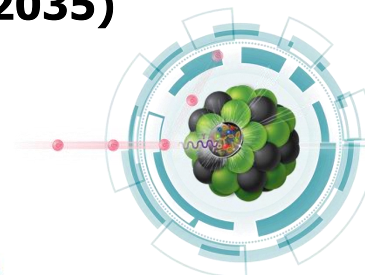
2D pressure anisotropy distribution

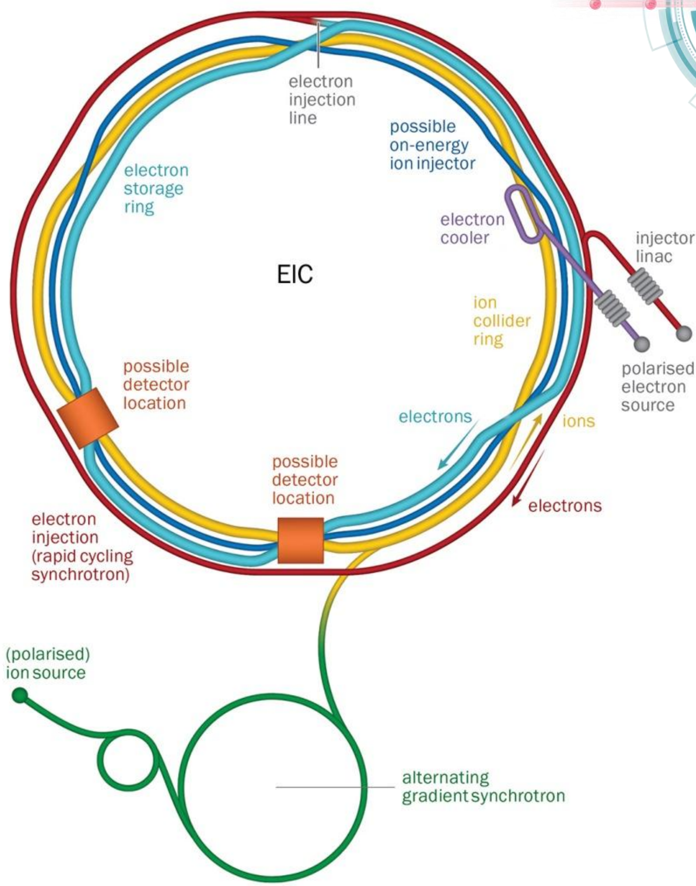


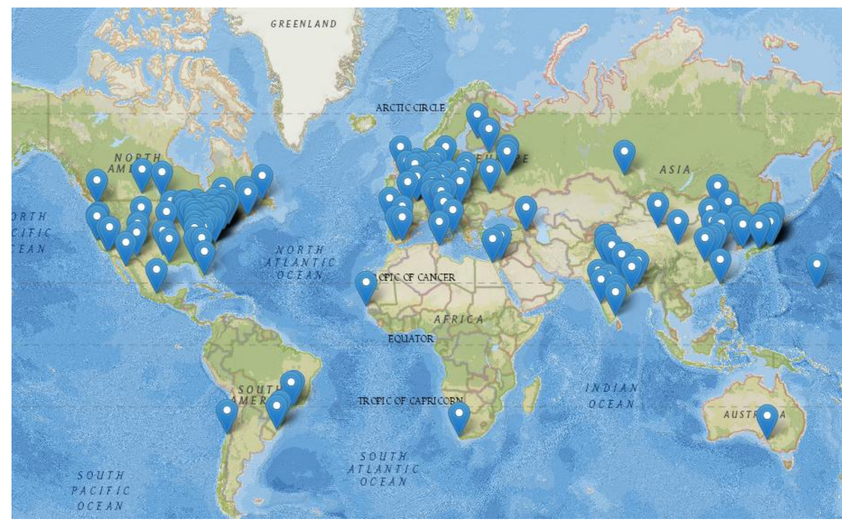
[Won, C.L., PRD111 (2025) 9, 09421]
[Won, C.L., PRD106 (2022) 1, 014012]

Key players in the near future

Electron-Ion Collider (~2035)
Brookhaven, Long Island







>1500 members from 290 institutions in 40 countries

[Abdul Khalek *et al.*, NPA1026 (2022) 122447]

but also JLab 20+ GeV, EicC, …

[Anderle *et al.*, FP16 (2021) 6, 64701]
[Accardi *et al.*, EPJA 60 (2024) 9, 173]

Some reviews

Hadron structure	[C.L., Metz, Pasquini, Schweiter, 2507.12664]
GPDs	[Diehl, PR388 (2003) 41] [Belitsky, Radyushkin, PR418 (2005) 1] [Kumericki, Liuti, Moutarde, EPJA52 (2016) 6, 157]
EMT & GFFs	[Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025] [Burkert *et al.*, RMP95 (2023) 4, 041002]
Spin decomposition	[Leader, C.L., PR541 (2014) 3, 163] [Wakamatsu, IJMPA29 (2014) 1430012] [Liu, C.L., EPJA52 (2016) 6, 379] [Ji, Yuan, Zhao, NRP3 (2021) 1, 27]
Mass decomposition	[Ji, FP16 (2021) 6, 64601] [C.L., Metz, Pasquini, Rodini, JHEP11 (2021) 121]

… and references therein !