Seminar, Center for Nuclear Femtography, Jefferson Lab, 10 Feb. 2026

Nucleon form factors of the energy-momentum tensor

Peter Schweitzer (UConn)

Outline

- introduction, definition, notation, brief overview about $D(t)$
- interpretation, 3D and 2D, pressure in quantum systems
- study in large-N_c mean field approach, 3D densities
- illustration in classical model with QED effects
- conclusions

based on • Goeke et al, PRD 75 (2007) 094021 • Neubelt et al, PRD101 (2020) 034013
• Polyakov, PS, Int.J.Mod.Phys.A 33 (2018) • Burkert et al, Rev.Mod.Phys. 95 (2023)
• M. Varma, PS, PRD102 (2020) 014047 • A. Mejia, PS, 2511.21916 [hep-ph]
• Lorcé, PS, Acta Phys.Polon.B 56 (2025) 3-A17

supported by • NSF grant no. 2412625, • DOE QGT Topical Collaboration

introduction

definition nucleon EMT form factors I (Ji 1996)

$$\langle N'|\hat{T}^{a}_{\mu\nu}|N\rangle = \bar{u}(p')\Bigg[A^a(t,\mu^2)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M} + \overline{C}^a(t,\mu^2)\,M\,g_{\mu\nu} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\Bigg]u(p)$$

- conserved current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ $(a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, etc, $\sum_a \overline{C}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $B(0) = 0$ $\Leftrightarrow$ total anomalous gravitomagnetic moment vanishes *

  D-term $\Leftrightarrow$ $D(0) \equiv \boldsymbol{D}$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* equivalent to: total nucleon spin $J^q + J^g = \frac{1}{2}$ is due to quarks + gluons (via Gordon identity)

definition nucleon EMT form factors II (Ji 1996)

$$\langle N'|\hat{T}^{a}_{\mu\nu}|N\rangle = \bar{u}(p')\Big[\, A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M} + J^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + \overline{C}^a(t,\mu^2)\,M\,g_{\mu\nu} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\,\Big]u(p)$$

- conserved current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ $(a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, etc, $\sum_a \overline{C}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $J(0) = \frac{1}{2}$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon spin *

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p'+p) \qquad \Delta = (p'-p) \qquad t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d^q_1(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M^q_2(t)$

* equivalent to: nucleons total anomalous gravitomagnetic moment vanishes (Gordon identity)

how we learn about hadrons

$|N\rangle$ = **strong**ly–interacting particle. Probe it with other forces! (easier approach)

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'|J^\mu_{\text{em}}|N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'|J^\mu_{\text{weak}}|N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'|T^{\mu\nu}_{\text{grav}}|N\rangle$	$\longrightarrow$	$A(t),\ B(t),\ D(t)$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

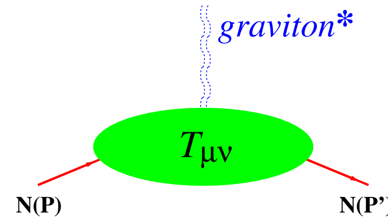
global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40) \times 10^{-19}\text{C} \\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N \\
g_A &= 1.2694(28) \\
g_p &= 8.06(0.55) \\
M &= 938.272013(23)\,\text{MeV} \\
J &= \tfrac{1}{2} \\
\boldsymbol{D} &= \textbf{?}
\end{aligned}
$$

see particle data book
(and more: form factors, PDFs, . . .

$\hookrightarrow$ ***D*** *least known global property*
not in particle data book (not yet)

define EMT and measure form factors



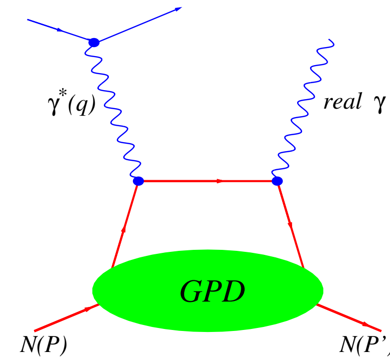
- **use gravity to define EMT**

$$\hat{T}_{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{grav}}}{\delta g^{\mu\nu}(x)}$$

- **use em to measure hard exclusive reactions**

Müller et al Fortsch. Phys. **42**, 101 (1994)
Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997)
Radyushkin, PLB **380**, 417, PLB **385**, 333 (1996)
Collins, Frankfurt, Strikman, PRD **56**, 2982 (1997)

$$\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] \boldsymbol{H_q(x,\xi,t)}$$



DVCS

- **polynomiality, fixed-t dispersion relation**

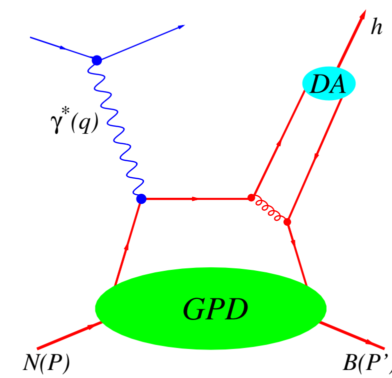
Ji 1996; Anikin & Teryaev 2008, Diehl & Ivanov 2007

$$\int \mathrm{d}x\; x\, \boldsymbol{H^q(x,\xi,t)} = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$$

$$\int \mathrm{d}x\; x\, \boldsymbol{E^q(x,\xi,t)} = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$$

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \mathrm{PV}\int_0^1 \frac{d\xi'}{\pi}\left[\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t)$$

$$\text{with } \Delta(t) \overset{\mu^2\to\infty}{\longrightarrow} 5\sum_q e_q^2\, \boldsymbol{D^q(t,\mu^2)} + \ldots$$



HMP

Why are EMT form factors interesting?

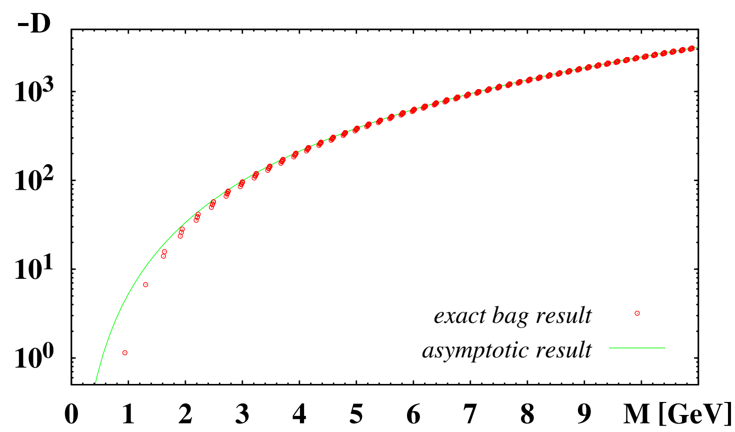
- proton mass decomposition $\rightsquigarrow$ $A^a(0)$, $\overline{C}^a(0)$ $\rightsquigarrow$ several workshops
  Ji 1995, Hatta et al 2018, Tanaka 2019, Metz et al, Rodini et al 2020, Lorcé et al 2021, Ji 2021, . . .

- proton spin decomposition $\rightsquigarrow$ $J^a(0)$ $\rightsquigarrow$ tens of workshops
  Ji 1996, many more works, Leader & Lorcé 2014, many works, Ji, Yuan, Zhao 2021, . . .

- D-term $\rightsquigarrow$ $D = D(0)$ $\rightsquigarrow$ so far 1 workshop, 1 in preparation
  Polyakov, Weiss 1999, slow starter, interest increasing (see below)

D-term theory overview, selected results

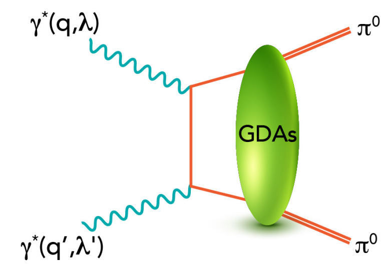
- **free spin-$\frac{1}{2}$ fermion** $D = 0$ Donoghue et al (2002), Hudson, PS (2018)
- **pions** Goldstone bosons of chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- **scalar particles in Φ^4** $D = -\frac{1}{3}$ Brean Maynard (2024)
  improvement term, Callan, Coleman, Jackiw (1970)
- **nuclei** (liquid drop model, Walecka model) $D \propto -A^{7/3}$
  Polyakov (2002), Guzey, Siddikov (2006); Liuti, Taneja (2005)
- **nucleon** bag model Ji, Melnitchouk, Song (1997), ...
  chiral quark soliton Goeke et al, PRD75 (2007), ...
- **χPT** Belitsky, Ji (2002), Alharazin, Djukanovic, Gegelia, Polyakov (2020), ...
  bound $D \leq -0.95(9)$ in chiral limit Gegelia, Polyakov (2021)
- **lattice QCD** Göckeler et al, PRL92 (2004), Shanahan, Detmold (2019), ...
  nucleon $D = -(3.87 \pm 0.97)$ Hackett, Pefkou, Shanahan (2024)
- **dispersion relations**
  $D = -2.66$ Pasquini, Polyakov, Vanderhaeghen (2014)
- **excited states** N^{th} excited Q-ball state: $M \propto N^3$ vs
  $D \propto N^8$ Mai, PS (2012); bag model Neubelt et al (2019)
- **reviews** Polyakov, PS, Int.J.Mod.Phys.A 33 (2018)
  Burkert et al, Rev.Mod.Phys. 95 (2023)



of all properties, D-term most sensitive to details of interaction & dynamics!

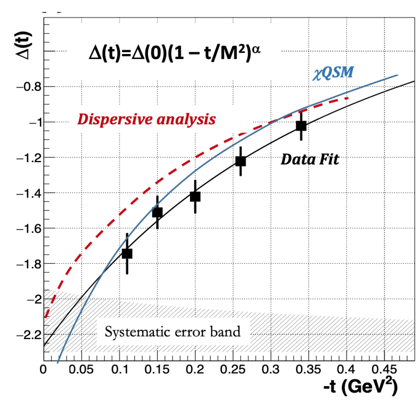
D-term overview, some phenomenological results

- D-term of π^0 from $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^- Belle, PRD, 2016
  generalized parton amplitudes (analytic continuation of GPDs)
  $D^Q_{\pi^0} \approx -0.7$ at $\langle Q^2 \rangle = 16.6$ GeV^2 Kumano, Song, Teryaev 2018

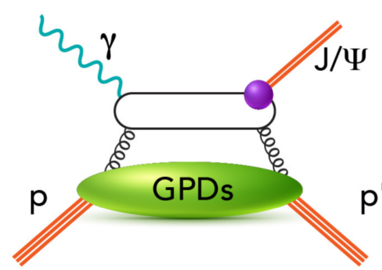


- proton GPD extraction from data not possible directly
  one cannot deconvolute $\text{CFF} = \text{Re}\mathcal{H}(\xi,t) + i\,\text{Im}\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t)$

- fixed-t dispersion relation JLab DVCS data
  $\mathcal{I}m\,\mathcal{H}$ $\rightsquigarrow$ BSA Girod et al PRL 100 (2008) 162002
  $\mathcal{R}e\,\mathcal{H}$ $\rightsquigarrow$ σ_{unp} Jo et al PRL 115 (2015) 212003
  $D_{u+d} = -1.63 \pm 0.11 \pm 0.26 \pm \ldots$ at $\langle Q^2 \rangle = 1.5\,\text{GeV}^2$
  Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)



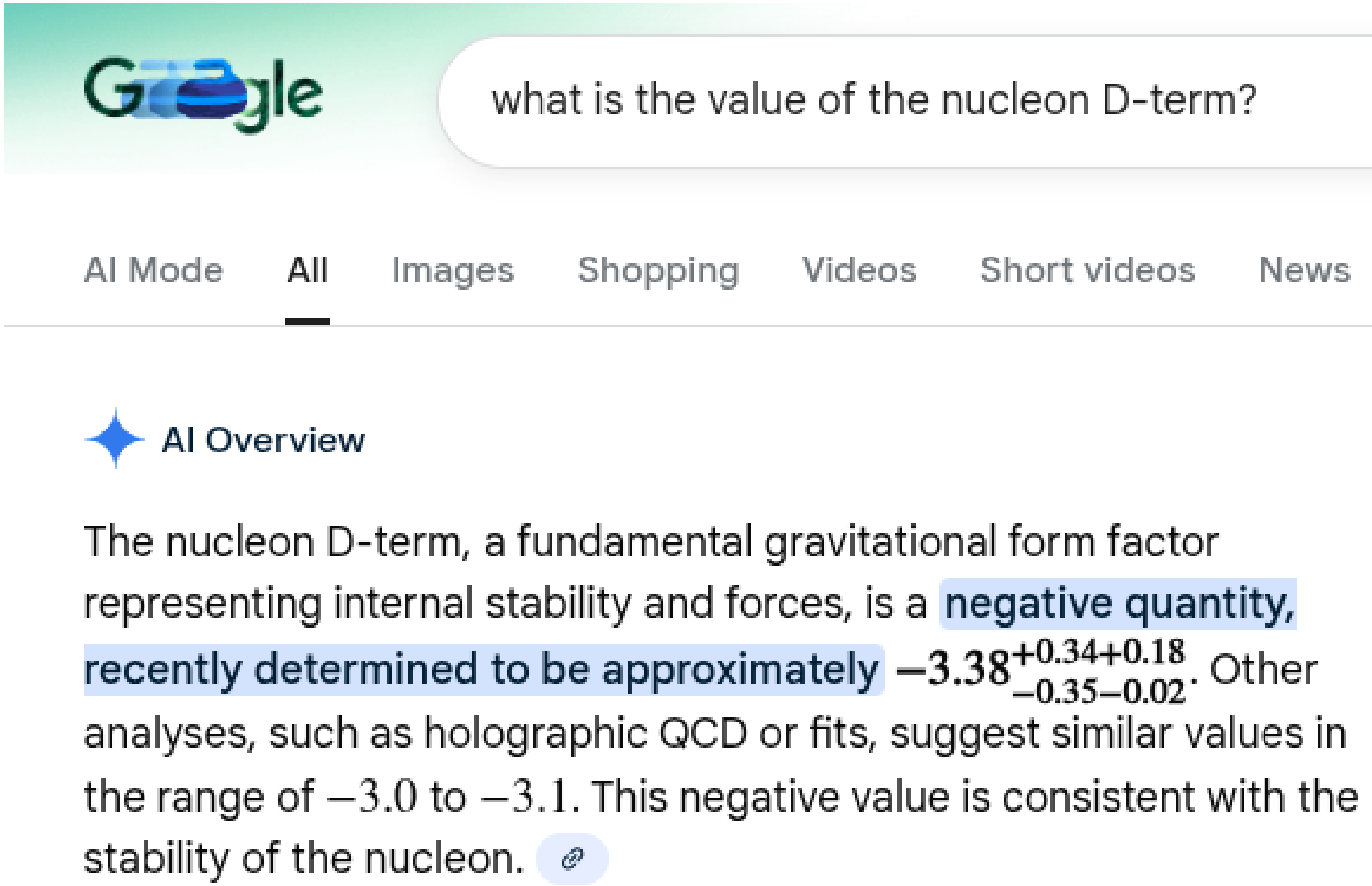
- gluon EMT form factors of proton
  $\rightsquigarrow$ threshold J/ψ photo-production
  Kharzeev 1995, 2021; Guo et al 2021
  Duran et al, Nature 615 (2023)



D-term remains least known global property, but now something is known

ask google:

Feb. 4, 2026



sources:

1. dispersive determination, Cao, Guo, Li, Yao, Nature Commun. 16 (2025) 6979
2. Yao, Xu, Binosi, Cui, Ding, Raya, Roberts, Rodríguez-Quintero, Schmidt, EPJA (2025) 61, 92
3. Fujii, Kawaguchi, Tanaka, PLB 866 (2025) 139559
4. Fujita, Hatta, Sugimoto, Ueda, Prog.Theor.Exp.Phys. (2022) 093B06
5. talk by PS(!) at EIC User Group Meeting 2019 in Paris, 22–26 July 2019
6. Broniowski, Ruiz Arriola, arXiv:2503.09297 [hep-ph] (published in PRD)
7. Goeke et al, PRD 75 (2007) 094021, chiral quark soliton
8. Hori, Suganuma, Kanda, PRD 109 (2024) 014030

interpretation

interpretation of EMT form factors

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Generalized parton distributions and strong forces inside nucleons and inside nuclei

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Received 2 December 2002; accepted 7 January 2003

Editor: J.-P. Blaizot

Abstract

We argue that generalized parton distributions (GPDs), accessible in hard exclusive processes, carry information about the spatial distribution of forces experienced by quarks and gluons inside hadrons. This way the measurements of hard exclusive processes open a possibility for direct "measurements" of strong forces in different parts of nucleons and nuclei. Also such studies open avenue for addressing questions of the properties of the quark (gluon) matter inside hadrons and nuclei. We give a simple example of relations between GPDs and properties of "nuclear matter" in finite nuclei.

step 1.

Let us now analyze the physics content of the form factors $M_2^Q(t)$, $J^Q(t)$ and $d^Q(t)$. To reveal the physics content of these form factors, in the same way as for electromagnetic form factors [17], it useful to consider the nucleon matrix elements of the energy–momentum tensor in Breit frame. In this frame the energy transfer $\Delta^0 = 0$, therefore, one can introduce the static energy–momentum tensor defined as:

$$T^Q_{\mu\nu}(\vec r,\vec s) = \int \frac{d^3\Delta}{(2\pi)^3\, 2E} e^{i\vec r\cdot\vec\Delta} \langle p',S'|\hat T^Q_{\mu\nu}(0)|p,S\rangle, \tag{5}$$

step 2.

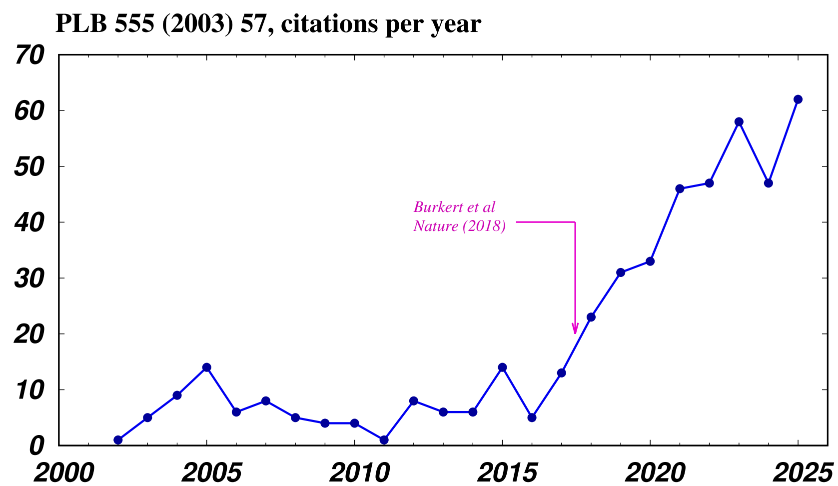
3. Now let us turn to the physics content of the form factor $d^Q(t)$. It is easy to see that this form factor is related to the traceless part of the static stress tensor $T^Q_{ik}(\vec r,\vec s)$ which characterizes the spatial distribution (averaged over time) of shear forces experienced by quarks in the nucleon [22]. In detail this relation is the following:

$$d^Q(t) + \frac{4}{3} t \frac{d}{dt} d^Q(t) + \frac{4}{15} t^2 \frac{d^2}{dt^2} d^Q(t) = -\frac{m_N}{2}\int d^3r\, e^{-i\vec r\cdot\vec\Delta}\, T^Q_{ij}(\vec r)\left(r^i r^j - \frac{1}{3}\delta^{ij} r^2\right). \tag{10}$$

If one considered the nucleon as a continuous medium then $T^Q_{ij}(\vec r)$ would characterize the force experienced by quarks in an infinitesimal volume at distance $\vec r$ from the centre of the nucleon. At $t = 0$ Eq. (10) gives:

$$d^Q(0) = -\frac{m_N}{2}\int d^3r\, T^Q_{ij}(\vec r)\left(r^i r^j - \frac{1}{3}\delta^{ij} r^2\right). \tag{11}$$

Maxim V. Polyakov



interest increasing! ☺

discussions

history of discussions around interpretations in physics

- interpretation of quantum mechanics $\longrightarrow$ since 100 years

  Max Born, 25. June 1926 (footnote added in proof: $|\psi|^2 \propto$ probability), ...

- 3D interpretation of em form factors $\longrightarrow$ since 70 years

  M. K. Rose, PR73, 279 (1948), Yennie, Lévy, Ravenhall, Rev.Mod.Phys. 29, 144 (publ. 1/1/1957)

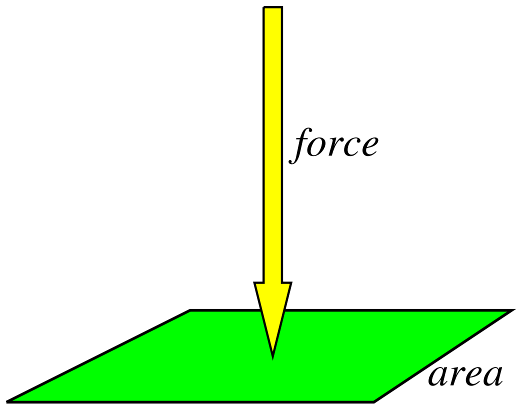
- interpretation of $D(t)$ $\longrightarrow$ since 5 years or so

  Jaffe, PRD103 (2021) 016017, G. Miller, Ji et al, ...

  $D(t)$ interpretation has both:

  (i) 3D aspect (because pressure)

  (ii) quantum aspect (of pressure)



- discussion around $D(t)$ interpretation

  review Lorcé, PS, Acta Phys.Polon.B 56 (2025) 3-A17 $\rightarrow$ DPP Memorial Volume

  3D aspect Gerry Miller, PRC (2025) $\rightarrow$ impossible to define 3D densities of rel. constituents

  quantum pressure Xiangdong Ji, Chen Yang [2508.16727] $\rightarrow$ journey seeking pressure in nucleon

2D partonic probability density interpretation vs ...

- proton made of particles → "spoiled" by parton model Feynman 1969

  requires infinite momentum frame or light-front quantization

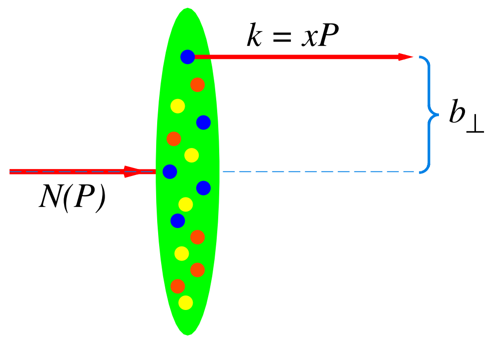
  light-front Fock expansion $|q, q, q\rangle + \ldots$ Brodsky, Miller, ...

  exact 2D **densities** → Matthias Burkardt (2000)

  2D EMT → Lorcé, Moutarde, Trawiński (2019); Freese, Miller (2021)

  3D partonic densities do not exist! Miller, PRC112 (2025) 045204

  if particle language, then: mean free path, forces between particles, etc → Ji, Yang



3D distribution interpretation

- proton made of fields (particle-wave duality for lack of a better word)

  $\langle N(p')|\hat{O}_{QM}|N(p)\rangle \stackrel{\text{F.T.}}{\longleftrightarrow}$ 3D **distributions**, unambiguously defined, intuitive, mechanical properties

  Breit frame interpretation justified

  - phase-space perspective → quasi-probabilistic interpretation Lorcé PRL 125 (2020) 232002
  - assume target is heavy, recoil corrections negligible (nuclei) Polyakov, PLB 555 (2003) 57
  - for nucleon rigorously justified in large N_c limit Goeke et al, PRD75 (2007) 094021

  unrelated to 2D and light-front wave functions Miller, PRC112 (2025) 045204

interpretation of EMT form factors

step 1

- **define static 3D EMT**

  in the Breit frame $\Delta^\mu = (0, \vec{\Delta})$
  M.V.Polyakov, PLB 555 (2003)

$$\boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$$

$$\int d^3r\; \boldsymbol{T_{00}(\vec{r})} = M \qquad \text{known}$$

$$\int d^3r\; \varepsilon^{ijk}\, s_i\, r_j\, \boldsymbol{T_{0k}(\vec{r})} = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5}M \int d^3r\, \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) \boldsymbol{T_{ij}(\vec{r})} \equiv \boldsymbol{D} \qquad \textbf{new!}$$

step 2

- nucleon extremely dense, continuous medium

$$\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}} \qquad \textbf{stress tensor}$$

$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **"mechanical properties"**

relation to stability, D-term, and forces

- EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$$\hookrightarrow \text{necessary condition for stability} \int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0 \quad \text{(von Laue, 1911)}$$

$$\boldsymbol{D} = -\frac{16\pi}{15}\, M \int_0^\infty \mathrm{d}r\; r^4 s(r) = 4\pi M \int_0^\infty \mathrm{d}r\; \boldsymbol{r^4\, p(r)} \rightarrow \text{internal forces}$$

relation to virial theorem

- variation of energy functional Goeke et al, PRD (2007), Lorcé, Metz, Pasquini, Rodini JHEP (2021)

$$\delta M \stackrel{!}{=} 0 \;\Leftrightarrow\; \int_0^\infty \mathrm{d}r\; \boldsymbol{r^2\, p(r)} = 0$$

eigenvalues of stress tensor

- symmetric 3×3 matrix $\rightarrow$ diagonalize Polyakov, PS, Int.J.Mod.Phys.A 33 (2018)

$\frac{2}{3}\, s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)

$-\frac{1}{3}\, s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$, degenerate for spin 0 and $\frac{1}{2}$)

mechanical radius

- **stability of a mechanical system**

  $\Leftrightarrow$ normal force directed towards outside

  $\Leftrightarrow T^{ij}e^j_r\,dA = \underbrace{[\frac{2}{3}\,s(r)+p(r)]}_{>0}e^i_r\,dA \quad \Rightarrow \quad \boldsymbol{D<0}$ Perevalova et al (2016)

- $$\langle \boldsymbol{r^2}\rangle_{\mathbf{mech}} = \frac{\int \boldsymbol{d^3r\ r^2[\frac{2}{3}\,s(r)+p(r)]}}{\int \boldsymbol{d^3r\ [\frac{2}{3}\,s(r)+p(r)]}} = \frac{\boldsymbol{6D(0)}}{\int_{-\infty}^{0}\mathbf{dt}\,\boldsymbol{D(t)}} \quad \text{vs} \quad \langle r^2_{\text{ch}}\rangle = \frac{6\,G'_{E,p}(0)}{G_{E,p}(0)} \quad \text{“anti-derivative”}$$

  intuitive result for large nucleus $\frac{2}{3}\,s(r)+p(r)=p_0\,\Theta(R_A-r) \ \to \ \langle r^2\rangle_{\text{mech}} = \frac{3}{5}\,R_A^2$

- proton: $\langle \boldsymbol{r^2}\rangle_{\mathbf{mech}} \approx \mathbf{0.75}\,\langle \boldsymbol{r^2_{\mathbf{ch}}}\rangle$ from chiral quark soliton model, Goeke et al 2007

- neutron: same $\langle r^2\rangle_{\text{mech}}$ as proton while $\langle r^2_{\text{ch}}\rangle_{\text{neut}} = -(0.11\,\text{fm})^2$ not particle size! (neglect QED)

- in chiral limit $\langle r^2\rangle_{\text{mech}}$ finite (!) vs $\langle r^2_{\text{ch}}\rangle$ divergent

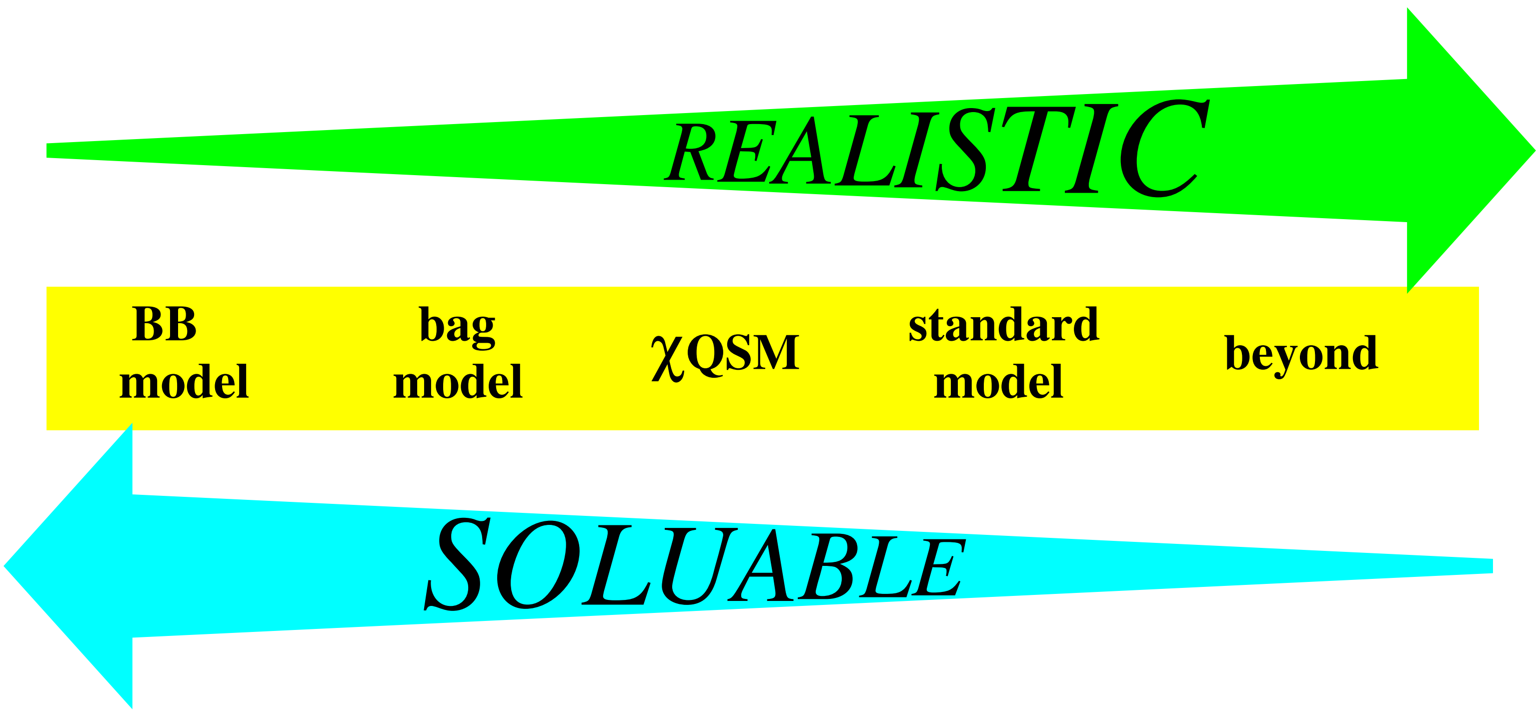
$\Rightarrow$ **mechanical radius better concept for particle size than electric charge radius**

due to $T_{00}(r)\geq 0$ also mass radius $\langle r^2_E\rangle = \int d^3r\,r^2T_{00}(r)/M = 6A'(0) - \frac{3D}{2M^2}$

Kharzeev, PRD 104, 054015 (2021)

illustration

disclaimer



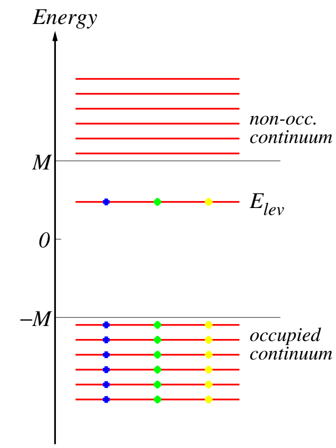
REALISTIC
BB model
bag model
χQSM
standard model
beyond
SOLUABLE

illustration in chiral quark-soliton model

- $\mathcal{L}_{\rm eff} = \overline{\Psi}\,(i\,\partial\!\!\!/ - M\,U^{\gamma_5})\Psi\,,\quad U = \exp(i\tau^a\pi^a/f_\pi)$
  Diakonov, Petrov, Pobylitsa, NPB 306, 809 (1988)

  solve in large-N_c limit, where $U(x) \to U(\vec{x})$ static mean field
  Witten NPB 160, 57 (1979)

  Hamiltonian $H = -i\gamma^0\gamma^i\nabla^i + \gamma^0 M\,U^{\gamma_5}$ with $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$
  spectrum discrete level and continua

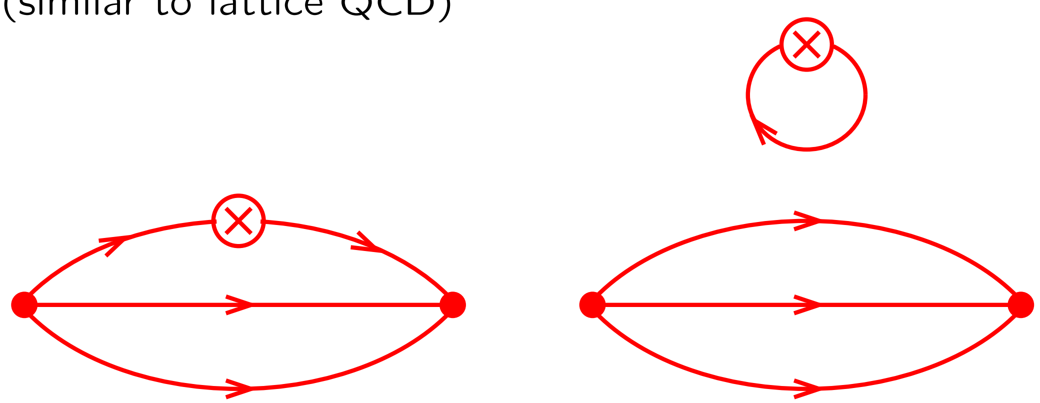


$$\langle N'|\hat{T}_{\mu\nu}|N\rangle = \lim_{T\to\infty}\frac{\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;\hat{T}_{\mu\nu}\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}{\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}$$

$$= 2M_N\int d^3x\; e^{i(\vec{p}\,'-\vec{p})\vec{x}}\,N_c\sum_{n,\rm occ}\overline{\Phi}_n(\vec{x})(\tfrac{1}{2}i\gamma^\mu\partial^\nu + \tfrac{1}{2}i\gamma^\nu\partial^\mu)\Phi_n(\vec{x}) + \dots$$

$$= 2M_N\int d^3r\; e^{i\vec{\Delta}\,\vec{r}}\,T_{\mu\nu}(\vec{r}) + \mathcal{O}(1/N_c^2) \quad\to\quad \text{3D spatial densities!}$$

connected & disconnected diagrams
(similar to lattice QCD)



$\sum_n \overline{\Phi}_n \dots \Phi_n \neq$ nucleon wave function(!)
but Greens function (quark field propagator in background mean field)

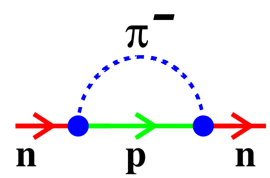
$G_{ij}(x,y) = i\langle 0|T\{\psi_i(y)\bar{\psi}_j(x)\}|0\rangle$
$= \langle y|\frac{1}{i\partial\!\!\!/ - MU^{\gamma_5}}|x\rangle$
$= i\Theta(x^0-y^0)\sum_{\rm non} e^{-iE_n(x^0-y^0)}\Phi_{n,i}(\vec{x})\bar{\Phi}_{n,j}(\vec{y})$
$\;\;-i\Theta(y^0-x^0)\sum_{\rm occ} e^{-iE_n(x^0-y^0)}\Phi_{n,i}(\vec{x})\bar{\Phi}_{n,j}(\vec{y})$

- We compute a three-point function in effective theory (not related to 2D light front densities) matrix element in momentum space = F.T.(3D distribution) in mean field approach
- **response functions!** Response to what? Insertion = stress tensor → mechanical properties response to ripples in metric, variations of g^{ik} ⇔ virial theorem
- soliton field time-independent in leading order of $N_c \to \infty$.
  do we miss dynamics, quantum fluctuations, loop corrections? → We miss nothing!
  model reproduces the leading non-analytic terms of spontaneous chiral symmetry breaking!

$$M(m_\pi) = M(0) + b_{M1}\, m_\pi^2 + b_{M2}\, m_\pi^3 + \dots \text{ with } b_{M2} = -\,k\,\frac{3g_A^2}{32\pi f_\pi^2}$$

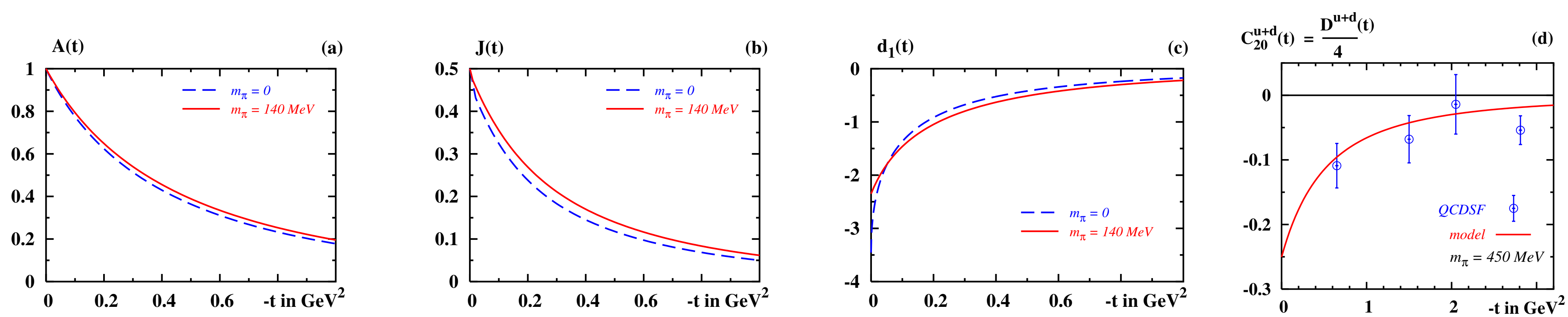
$$D(m_\pi) = D(0) + b_{D1}\, m_\pi + \dots \text{ with } b_{D1} = k\,\frac{3g_A^2 M}{16\pi f_\pi^2}$$

$$D(t) = D(0) + b_{D2}\sqrt{-t} + \dots \text{ with } b_{D2} = k\,\frac{9g_A^2 M}{128\pi f_\pi^2}$$



$\sqrt{-t}$ and $m_\pi \propto \sqrt{m_q}$ are non-analytic.
leading non-analytic terms model independent!
technical: $k = 1$ for N_c finite, $k = 3$ for $N_c \to \infty$

- concern: $N_c = 3$ not large. True. But large enough!
  $\frac{1}{N_c}$ the only small parameter of QCD at all energies.
  Witten 1979, Dashen, Jenkins, Manohar PRD 1994



Goeke et al, PRD75 (2007) 094021, Göckeler et al, NPB Proc.Suppl. 128 (2004) 2031
recall $d_1(t) = \frac{4}{5}D(t)$ and $C_{20}(t) = \frac{1}{4}D(t)$

visualization of forces

- **liquid drop model for large nuclei**

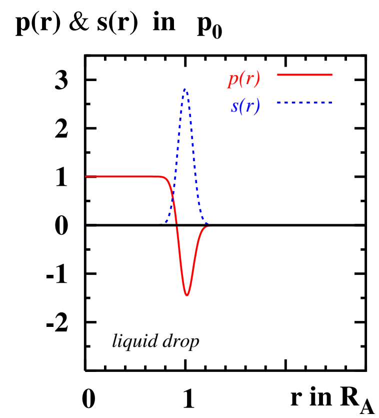
  $p(r) = p_0\,\Theta(R_A - r) - \frac{1}{3}p_0 R_A\,\delta(r - R_A)$, $s(r) = \gamma\,\delta(r - R_A)$

  $R_A = R_0\,A^{1/3}$, $m_A = m_0\,A$, surface tension $\gamma = \frac{1}{2}p_0 R_A$

  D-term $\boldsymbol{D = -\frac{4\pi}{3}\,m_A\,\gamma\,R_A^4 \approx -0.2\,A^{7/3}}$

  M.V.Polyakov PLB555 (2003)

  Guzey, Siddikov (2006); Liuti, Taneja (2005)

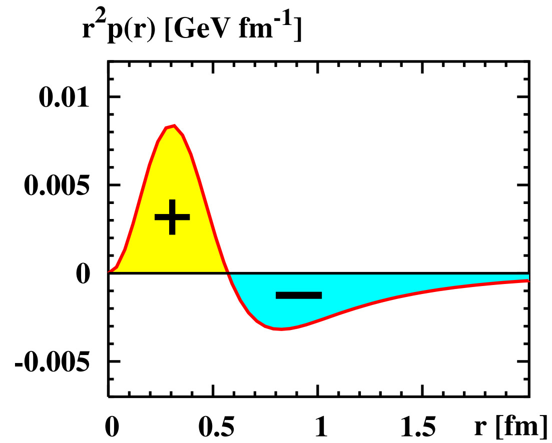


- **chiral quark soliton model**

  Goeke et al, PRD75 (2007) 094021

  $p(r) > 0$ for $r > r_0 = 0.57\,\text{fm}$,

  $p(r) < 0$ for $r < r_0$

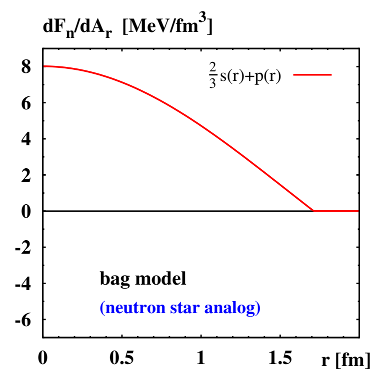


- **bag model**

  Neubelt et al PRD101 (2020) 034013

  normal force directed towards outside

  $dF_n^i = T^{ij} e_r^j\,dA_r = \underbrace{[\frac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\,dA_r$



very similar results in other hadronic approaches with short-range interactions

electromagnetic effects in proton

- does em interaction even matter for hadron structure?
- to find out we would like to find a model which
  (i) includes strong **and** em interaction,
  (ii) is theoretically consistent,
  (iii) solvable,
  (iv) lucid

excellent choice: classical proton model by Białynicki-Birula

Phys.Lett.A 182 (1993) 346-352

- no interpretation of form factors $\rightarrow$ 3D distributions
  instead compute distributions $\rightarrow$ predict form factors for $(-t) \neq$ large

- of course, the interior of hadrons requires a quantum description
  but we shall find pleasant surpizes!

Remark: since e^- discovery in 1897, many attempts to describe classical extended electrically charged particles by Thomson, Abraham, Lorentz, Poincaré, Einstein, Wien, Planck, Sommerfeld, Langevin, Ehrenfest, Born, Pauli, von Laue, ... Even after advent of QM and QFT → interesting theoretical problem for its own sake, Dirac (1962), Schwinger (1983), Białynicki-Birula (1993), ...

relativistic classical proton model

Białynicki-Birula, Phys.Lett.A 182 (1993)

"dust particles" described by phase-space distribution $\Gamma(\vec{r}, \vec{p}, t)$

- attract each other due to **strong massive scalar field** ϕ
- repel due to **strong massive vector field** V^μ, and
- due to **electromagnetic force due to** A^μ

field equations:

$$\left[(m - g_S\phi)(\partial_t + \vec{v}\cdot\vec{\nabla}_r) + m\,\vec{F}\cdot\vec{\nabla}_p\right]\Gamma(\vec{r}, \vec{p}, t) = 0$$

$$\partial_\alpha G^{\alpha\beta} + m_V^2 V^\beta = g_V\, j^\beta$$

$$(\Box + m_S^2)\phi = g_S\,\rho$$

$$\partial_\alpha F^{\alpha\beta} = e\, j^\beta$$

definitions:

$$\vec{F} = \vec{f}/u^0,$$

$$\rho(\vec{r}, t) = \int \frac{d^3p}{E_p}\, m_{\text{dust}}\, \Gamma(\vec{r}, \vec{p}, t)$$

$$j^\alpha(\vec{r}, t) = \int \frac{d^3p}{E_p}\, p^\alpha\, \Gamma(\vec{r}, \vec{p}, t)$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha, \quad G^{\alpha\beta} = \partial^\alpha V^\beta - \partial^\beta V^\alpha$$

with $p^\alpha = m\,u^\alpha$, $\vec{v} = \vec{u}/u^0$, and $f^\alpha = eF^{\alpha\beta}u_\beta + g_V G^{\alpha\beta}u_\beta - g_S(\partial^\alpha - u^\alpha u^\beta\partial_\beta)\phi$ (4-force acting on dust)
fully covariant, generalization of Vlasov-Maxwell equations in plasma physics
Białynicki-Birula, Hubbard, Turski, Physica 128A (1984) 504

proton solution

static in rest frame $u^\alpha = (1,0,0,0)$ with $\Gamma(\vec{r},\vec{p},t) = \delta^{(3)}(\vec{p})\,\rho(r)$

$$
\begin{aligned}
\rho(r) &= \Big(f_+(r) - f_-(r)\Big)\,\Theta(R_p - r)\,, \\
eA_0(r) &= e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2} + \frac{2E_B}{e^2}\right)\Theta(R_p - r) \;+\; \frac{e^2}{4\pi r}\,\Theta(r - R_p)\,, \\
g_S\phi(r) &= g_S^2\left(\frac{f_+(r)}{k_+^2 + m_S^2} - \frac{f_-(r)}{k_-^2 + m_S^2}\right)\Theta(R_p - r) \;+\; \frac{b_S}{4\pi r}e^{-m_S(r-R_p)}\,\Theta(r - R_p)\,, \\
g_V V_0(r) &= g_V^2\left(\frac{f_+(r)}{k_+^2 + m_V^2} - \frac{f_-(r)}{k_-^2 + m_V^2}\right)\Theta(R_p - r) \;+\; \frac{b_V}{4\pi r}e^{-m_V(r-R_p)}\,\Theta(r - R_p)\,,
\end{aligned}
$$

$f_\pm(r) = \frac{d_\pm \sin(k_\pm r)}{4\pi r}$, $k_\pm = \sqrt{\frac{B_p \pm \sqrt{D_p}}{2Q_p^2}}$, $B_p = (g_S^2 - e^2)m_V^2 - (g_V^2 + e^2)m_S^2$, $D_p = B_p^2 - 4e^2Q_p^2 m_S^2 m_V^2$,
$Q_p^2 = g_V^2 + e^2 - g_S^2$ with b_V, b_S, d_+, d_-, $2E_B$, R_p fixed from cont. and diff. of fields

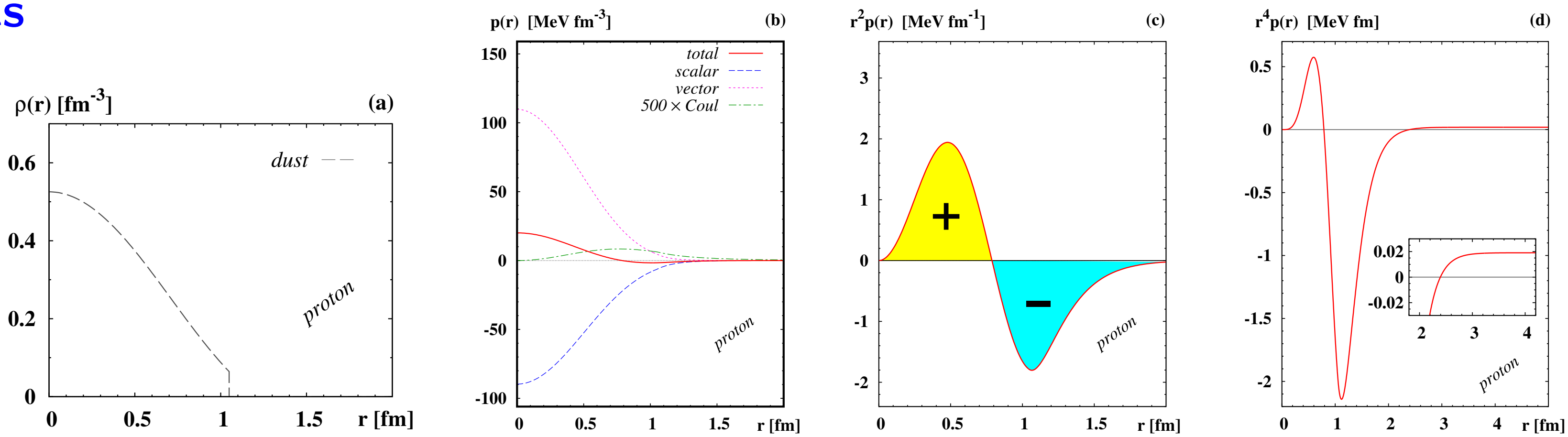
parameters $m_{\text{dust}} = 938\,\text{MeV}$

$$
m_S = 550\,\text{MeV}, \quad m_V = 783\,\text{MeV}, \quad \frac{g_S^2}{4\pi\hbar c} = 7.29\,, \quad \frac{g_V^2}{4\pi\hbar c} = 10.8\,, \quad \frac{e^2}{4\pi\hbar c} = \frac{1}{137}
$$

from mean field theory of nuclear matter model QHD-I

Serot, Walecka, Adv. in Nucl. Phys. Vol. 16 (1986)

- **mass** $M = m_{\text{dust}} + E_B$ with $E_B = -15.71\,\text{MeV}$ cf. Serot, Walecka, op. cit.
- **size** $\langle r^2_{\text{dust}}\rangle^{1/2} = 0.71\,\text{fm}$ vs proton charge radius $\langle r_p^2\rangle^{1/2}_{\text{ch}} = 0.84\,\text{fm}$
- **proton made of dust bound by residual nuclear forces**
- **results**



$p(0)$ in proton center $\sim 10\times$ less than χQSM (residual forces)

balance of strong opposite forces inside proton $\int_0^\infty dr\; r^2 p(r) = 0$

at long-distances $s(r) = -\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots$ & $p(r) = \frac{\alpha}{24\pi}\,\frac{\hbar c}{r^4} + \dots$ due to $T^{ik}_{\text{Maxwell}} = -\frac{1}{4\pi}(E^i E^k - \frac{1}{2}\delta^{ik}\vec{E}^2)$

- **consequence**

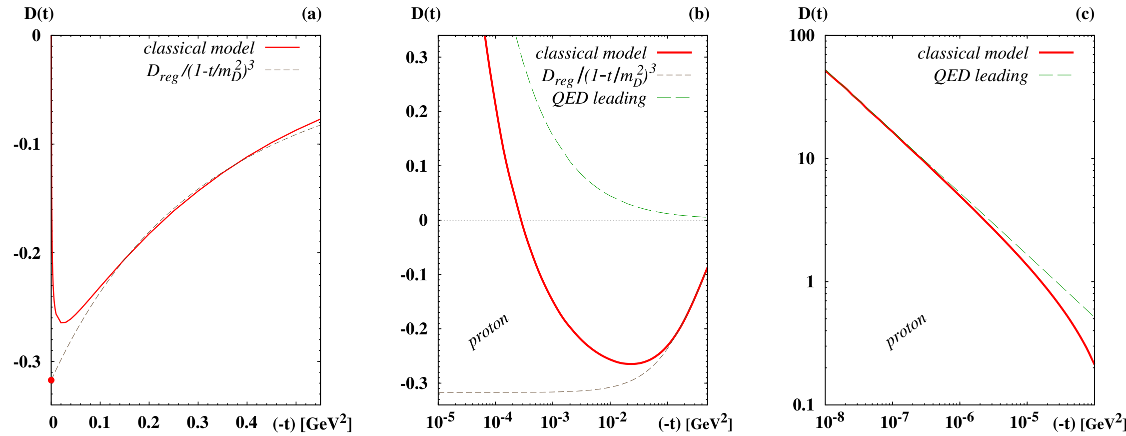
$D_s = -\frac{4}{15}\,M\int d^3r\; r^2 s(r)$ and $D_p = M\int d^3r\; r^2 p(r)$ diverge

$D(t) = \frac{\alpha\pi}{4}\,\frac{M}{\sqrt{-t}} + \dots$ as $t \to 0$

Berends, Gastmans (1976), Milton (1977), Kubis, Meissner (2000), Donoghue et al (2002), Metz et al (2021), Freese et al (2022)

...so proton has no D-term? $D_{\text{prot}} = +\infty$ ....??

$D(t)_{\text{prot}}$ and $D_{\text{prot,reg}}$



- if it exists, $D = \zeta\, D_p + (1-\zeta)\, D_s$
  same result for any value of ζ

- for proton divergent for all values except $\zeta = \frac{8}{3}$ $\rightarrow$ then $\frac{1}{r^4}$-tails in $p(r)$ and $s(r)$ exactly cancel

- regularized result $D_{\text{prot, reg}} = -0.317$ M. Varma, PS, PRD102 (2020) 014047

- **check** regularization in (neutron model) $= \lim\limits_{e\to 0}$(proton model)

- fascinating results

  neutron size (dust distribution) $\langle r^2_{\text{dust}}\rangle^{1/2} = 0.704\,\text{fm}$ vs proton $0.710\,\text{fm}$
  neutron slightly smaller and denser, proton swollen due to Coulomb repulsion

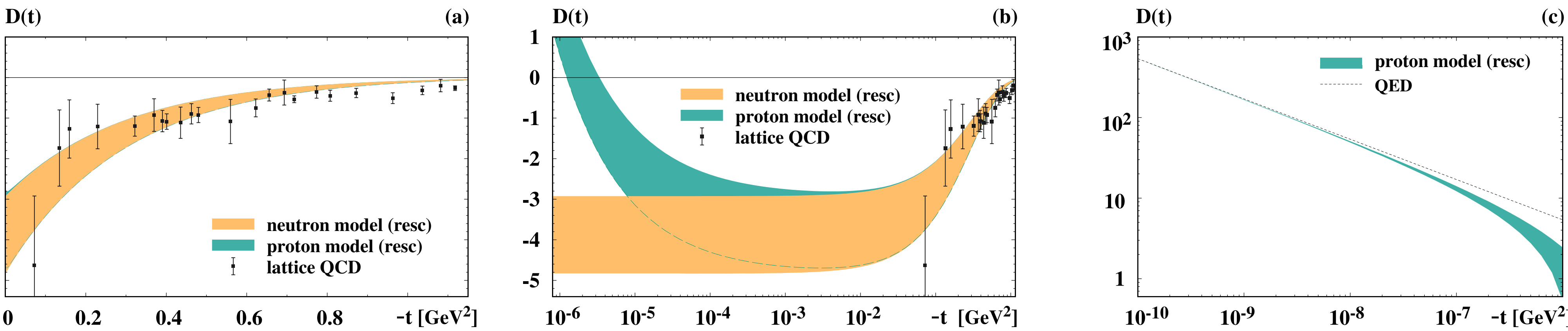
  electromagnetic mass difference
  classical model: $\boldsymbol{(M_p - M_n)_{em} = 0.95}$ **MeV**
  vs lattice QCD+QED: 1.00(07)(14) MeV
  Borsanyi et al, Science 347 (2015) 1452

  $\boldsymbol{D_{\text{neut}} = -0.312}$ nearly identical to $D_{\text{prot,reg}}$ (as one would hope)

Nice but unrealistic, proton charge radius 15 % and D-term order of magnitude too small

realistic model description of $D(t)$

- $m_i \to 0.85\, m_i$ to get the charge radius $\langle r_p^2 \rangle_{\text{ch}}^{1/2} = 0.83\,\text{fm}$
  in agreement $(0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}})$fm Xiong et al, Nature 575 (2019)
- $g_i \to (3.2 \pm 0.4)\, g_i$ to scale up $D(t)$ by about factor 10
  in agreement with lattice QCD Hackett, Pefkou, Shanahan, PRL (2024)



  - ▶ for $(-t) \gtrsim 10^{-4}\,\text{GeV}^2$ proton and neutron look the same
  - ▶ for $(-t) \lesssim 10^{-7}\,\text{GeV}^2$ proton approaches QED asymptotics

conclusion: until DVCS experiments can reach below $(-t) \approx 10^{-4}\,\text{GeV}^2$ with precision, $D(t)$ of proton and neutron will look finite and alike without visible QED effects!

keep in mind: $D(t)_{\text{prot}} \approx D_{\text{prot,reg}}/(1 - t/m_D^2)^N$ good approximation, $D_{\text{prot,reg}}$ good concept. But ultimately, $D(t)_{\text{prot}} \propto 1/\sqrt{-t}$ like charged pions, electron. Universal QED effect.

Rescaled model reproduces QED $(t \to 0)$ and lattice QCD $(0.08 < (-t) < 1\,\text{GeV}^2)$, i.e. **all** presently available first-principle constraints. Mejia, PS, 2511.21916 [hep-ph]

conclusions

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- **EMT form factors** $\rightarrow$ important
  mass & spin decompositions + D-term!
- **measurable** $A(t)$ and $B(t)$ hard (cannot invert CFF)
  $\boldsymbol{D(t)}$ from fixed-t dispersion relation more direct access
- **$\boldsymbol{D}$-term** least known global property, for fermions generated dynamically,
  negative: Goldstone bosons, models, dispersion relations, lattice QCD, experiment
- **2D interpretation** exact probability density interpretation
  infinite momentum frame or light-front quantization
- **3D interpretation** exact in mean field approach, large N_c QCD (pressure is 3D)
  response functions, momentum or coordinate space, distributions $\neq$ probability density
- **illustration in $\boldsymbol{\chi}$QSM** based on short-range chiral strong forces,
  mechanical stability, D negative, visualization of internal forces
- **illustration in classical BB model with em interaction**, simple, lucid model
  D of charged particles undefined (known before, community unaware)
- **proton in realistic rescaled BB-model** $\rightarrow$ reproduces QED and QCD results
  experimentally proton and neutron the same. QED effects noticeable for $(-t) \ll 10^{-4}\text{GeV}^2$
- **many more lessons** to learn about hadrons from experiment, theory, models
  through EMT form factors
- discussions of **mechanical properties** will likely continue, which is good(!)
  because we will improve the foundations of our understanding

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Thank you!

Support slides

D-term in theory

- **free spin-0 particle** $D = -1$
  Pagels 1966; Hudson, PS 2017

- **free spin $\frac{1}{2}$ particle** $D = 0$
  Donoghue et al, (2002), Hudson, PS PRD97 (2018) 056003

- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

$$
\begin{aligned}
D_\pi &= -1 + 16\, a\, \frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\, I_\pi - \frac{m_\pi^2}{3F^2}\, I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\, a\, \frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\, I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\, a\, \frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\, I_\pi + \frac{8m_K^2}{3F^2}\, I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\, I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\left(\log\frac{\mu^2}{m_i^2} - 1\right)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$
Donoghue, Leutwyler (1991)
estimates: Hudson, PS (2017)

- **nuclei** (liquid drop model, Walecka model) $\boldsymbol{D \approx -0.2 \times A^{7/3}}$ $\rightarrow$ DVCS with nuclei!
  Polyakov (2002),
  Guzey, Siddikov (2006);
  Liuti, Taneja (2005)

$$
\begin{aligned}
{}^{12}\text{C}: \quad & D = -6.2 \\
{}^{16}\text{O}: \quad & D = -115 \\
{}^{40}\text{Ca}: \quad & D = -1220 \\
{}^{90}\text{Zr}: \quad & D = -6600 \\
{}^{208}\text{Pb}: \quad & D = -39000
\end{aligned}
$$

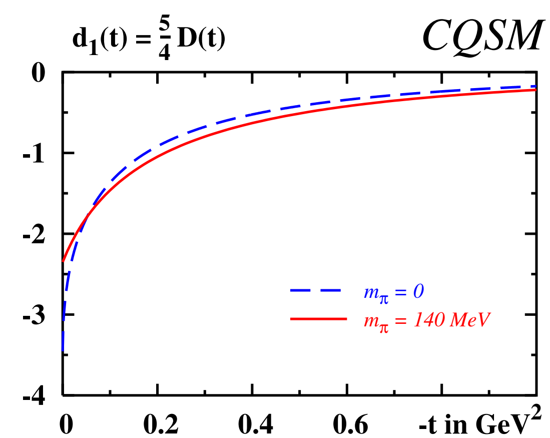
- $\boldsymbol{Q}$**-balls** N^{th} excited Q-ball state: mass $M \propto N^3$ but $\boldsymbol{D \propto N^8}$
  Mai, PS PRD**86**, 096002 (2012)

- **nucleon, bag model** $\boldsymbol{D = -1.15 < 0}$
  Ji, Melnitchouk, Song (1997)

- **chiral quark soliton**
  Goeke et al, PRD75 (2007)

$$d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5\,k\,g_A^2 M}{64\,\pi\,f_\pi^2}\,m_\pi + \dots$$

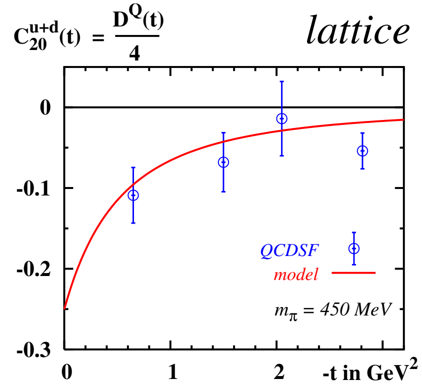
$$\overset{\circ}{d}{}_1'(0) = -\frac{k\,g_A^2\,M}{32\,\pi f_\pi^2\,m_\pi} + \dots \quad k = \begin{cases} 1, & N_c \text{ finite} \\ 3, & N_c \to \infty \end{cases}$$



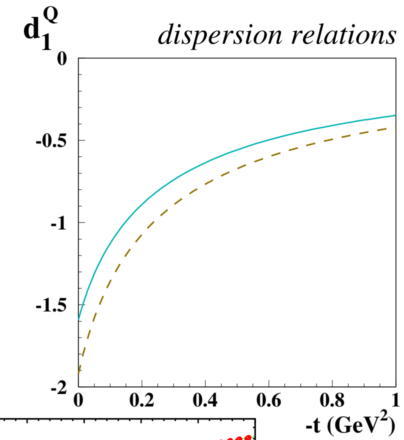
- χ**PT**
  Belitsky, Ji (2002), Diehl et al (2006),
  Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023

- **non-relativistic limit** $\boldsymbol{D = -N_c^2\,\frac{4\pi^2-15}{45}} = -4.89$
  Neubelt et al (2019) (in bag)

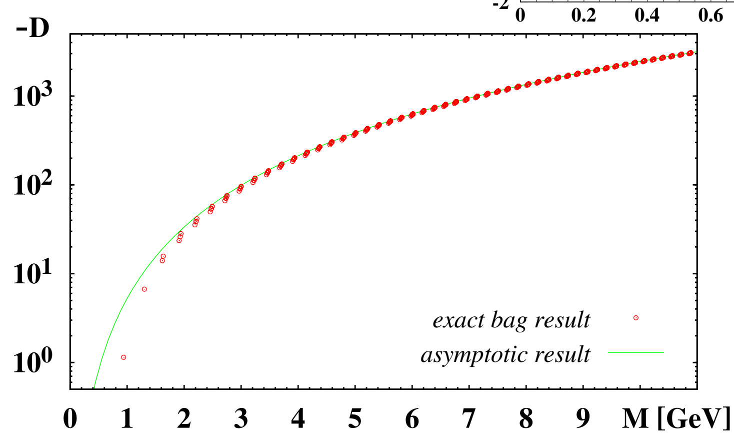
- **lattice:** QCDSF
  Göckeler et al, PRL92 (2004)
  $\mu = 2\,\text{GeV}$, $m_\pi = 450\,\text{MeV}$
  disconnected diagrams neglected
  recently:
  $D^g(t) < 0$ with $|D^g(t)| > |D^Q(t)|$
  Shanahan, Detmold, PRD99 (2019)



- **dispersion relations** $d_1^Q(t) = \frac{5}{4}\,D^Q(t)$
  Pasquini, Polyakov, Vanderhaeghen (2014)
  pion PDFs are input, scale $\mu^2 = 4\,\text{GeV}^2$



- **excitated stats**
  in bag model Neubelt et al (2019)
  M over 1 order of magnitude
  D over 3 orders of magnitude



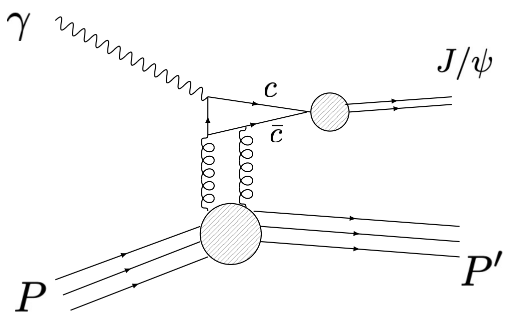
of all properties, D-term most sensitive (parameters, excitations)

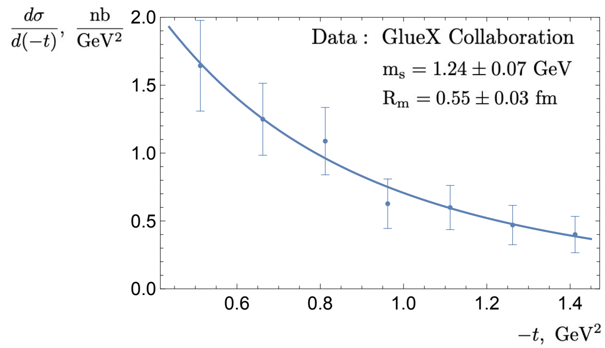
$\Rightarrow$ dynamics!

keep in mind: free spin $\frac{1}{2}$ theory $\rightarrow D = 0$;

i.e. D-term of nucleon due to dynamics!

- **form factor of** $\hat{T}_\mu^{\ \mu} = \frac{\beta(g)}{2g} F^2 + \mathcal{O}(m_q)$ from J/ψ photoproduction at threshold
Hatta 2019, Kharzeev 2021





Gluex PRL 123, 072001 (2019).

$\sqrt{\langle r^2_{\text{trace}} \rangle} = 0.55 \pm 0.03$ fm < charge radius ~ 0.84 fm

$\sqrt{\langle r^2_{\text{traceless}} \rangle_g} \sim (0.3\text{–}0.35)$ fm of $A^g(t) = A^g(0) + \frac{1}{6}\, t\, \langle r^2_{traceless} \rangle_g + \ldots$ from QCD sum rules
Braun, Górnicki, Mankiewicz, Schäfer, PLB 302, 291 (1993)

explanation:
$\langle r^2_{\text{trace}} \rangle_g$ due to one-instanton contributions, vs $\langle r^2_{\text{traceless}} \rangle_g$ from instanton-anti-instanton
i.e. suppressed by instanton packing fraction Diakonov, Polyakov, Weiss (1996)

relation to other EMT form factors:

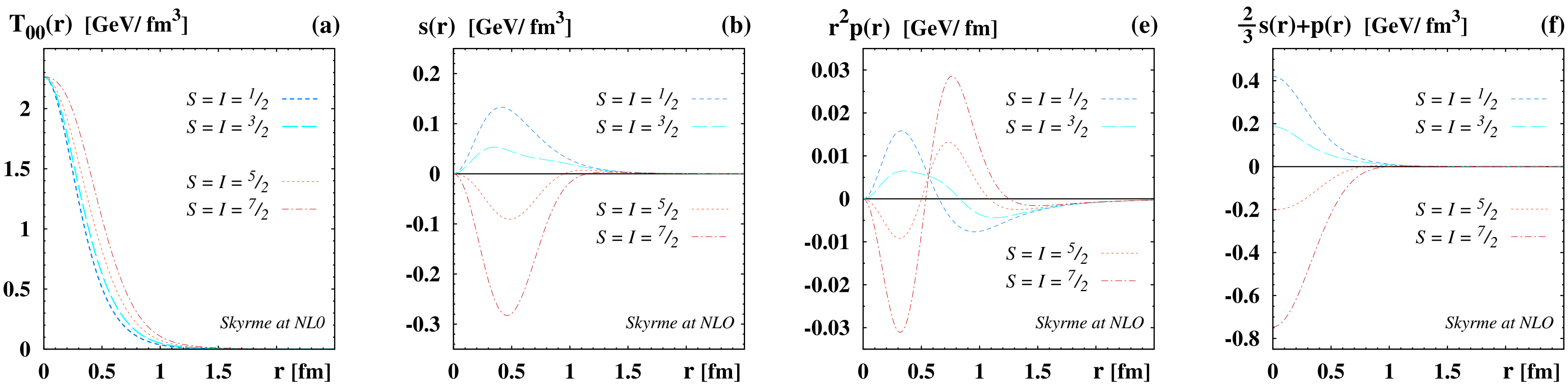
form factor $\langle p' | \hat{T}^\mu{}_\mu | p \rangle = \bar{u}(p') u(p)\, F_{\text{tr}}(t)$ where $F_{\text{tr}}(t) = 1 + \frac{1}{6}\, t\, \langle r^2_{\text{tr}} \rangle + \mathcal{O}(t^2)$

$$F_{\text{tr}}(t) = A(t) + \frac{t}{4M^2} B(t) - \frac{3t}{4M^2} D(t) = 1 + t\left(\frac{dA(0)}{dt} - \frac{3D}{4M^2}\right) + \mathcal{O}(t^2)$$

$\langle r^2_{\text{trace}} \rangle = 6\, A'(0) - \frac{9D}{2M^2}$ "mass radius"

Skyrme model nucleon, Δ vs large-N_c artifacts Witten 1979

- in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

stability requires:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

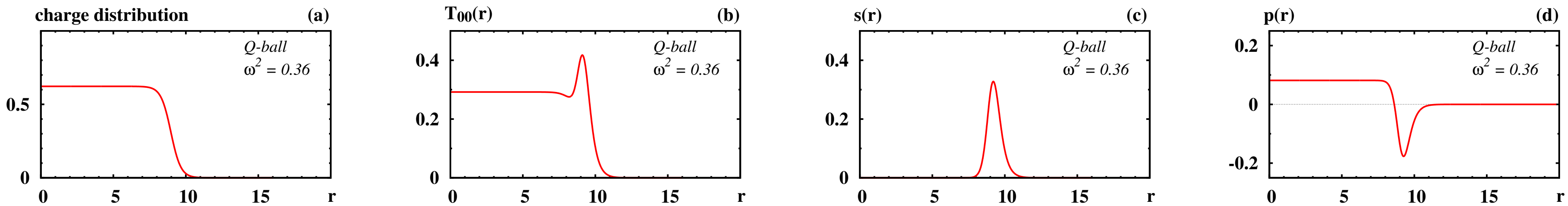
mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \ \frac{2}{3}s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ have positive D-term!
So do not exist!
dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Q-balls $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$, $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$

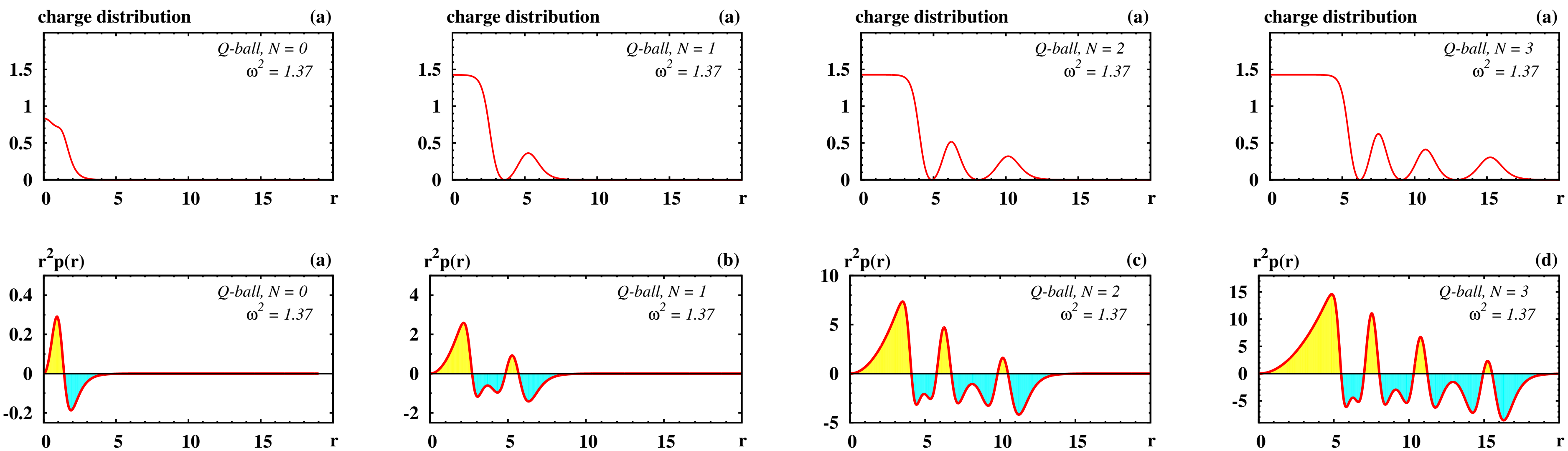
global U(1) symmetry, solution $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

- ground state properties for large Q-ball



- excitations: $N=0$ ground state, $N=1$ first excited state, etc Volkov, Wohnert 2002; Mai, PS 2012

charge distribution exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros

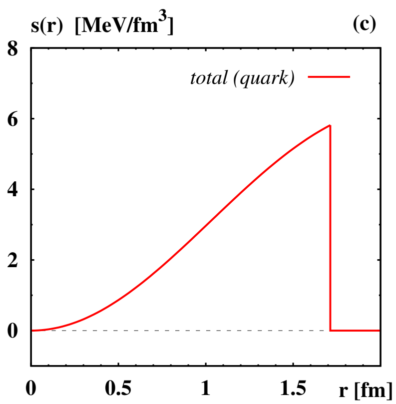


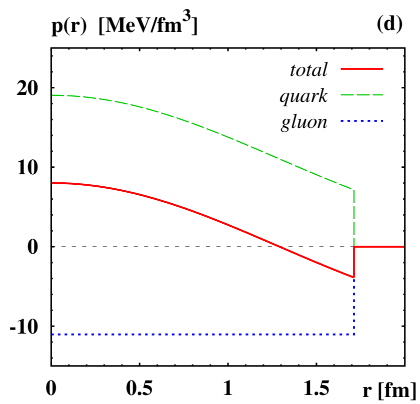
excited states unstable, but $\int_0^\infty dr\, r^2 p(r) = 0$ always valid, and D-term always negative!

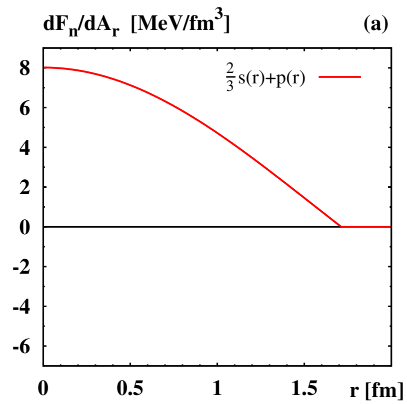
bag model Neubelt, Sampino, Hudson, Tezgin, PS, PRD101 (2020) 034013

- free quarks + boundary condition, formulated in large-N_c
- $T^{\mu\nu}(r) = T^{\mu\nu}_{\text{quarks}}(r) + T^{\mu\nu}_{\text{bag}}(r)$

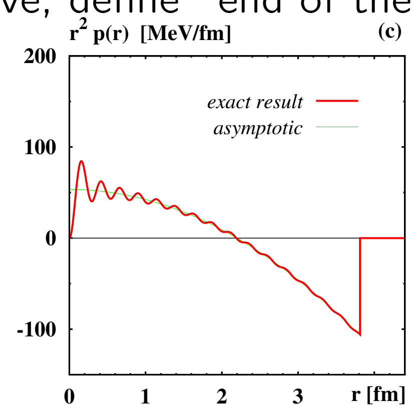
  $T^{\mu\nu}_{\text{bag}}(r) = B\,\Theta(R-r)\,g^{\mu\nu}$ binding effect ("mimics gluons" Jaffe & Ji 1991)
- all distributions defined with Θ-functions, assume non-zero values at $r = R$







- only exception:
  the normal force $= \frac{2}{3}\,s(r) + p(r) > 0$ for $r < R$, becomes exactly zero at $r = R$
- this is how one determines the radius of a neutron star:
  solve Tolman-Oppenheimer-Volkoff equations with an "equation of state"
  where "radial pressure" $\frac{2}{3}\,s(r) + p(r)$ turns negative, define "end of the system"
- excitated states different pattern than Q-balls:
  $p(r)$ has one node (here 3163th excited state)
  but $D \sim \text{const} \times M^{8/3}$ bag & Q-balls
  deeper reason?



D-term in the presence of long-range forces

Simple relativistic classical model of a finite size particle Białynicki-Birula, Phys. Lett. A 182 (1993) 346

non-interacting "dust particles" within R described by phase-space distribution $\Gamma(\vec{r},\vec{p},t)$ feel 3 forces:

- massive scalar field force (attractive, mass m_S, short range $\sim \frac{1}{r}\, e^{-m_s r}$)
- massive vector field force (repulsive, mass $m_V > m_S$, even shorter range $\sim \frac{1}{r}\, e^{-m_V r}$)
- massless vector field force (repulsive, Coulomb force, infinite range $\sim \frac{1}{r}$)

$$\begin{aligned}
\Big[(m-g_S\phi)(\partial_t+\vec{v}\cdot\vec{\nabla}_r)+m\,\vec{F}\cdot\vec{\nabla}_p\Big]\,\Gamma(\vec{r},\vec{p},t) &= 0\,,\\
\partial_\alpha G^{\alpha\beta}+m_V^2V^\beta &= g_V\,j^\beta,\\
(\Box+m_S^2)\phi &= g_S\,\rho\,,\\
\partial_\alpha F^{\alpha\beta} &= e\,j^\beta,
\end{aligned}$$

with $j^\alpha(\vec{r},t)=\displaystyle\int\frac{d^3p}{E_p}\,p^\alpha\,\Gamma(\vec{r},\vec{p},t),\quad \rho(\vec{r},t)=\int\frac{d^3p}{E_p}\,m\,\Gamma(\vec{r},\vec{p},t).$ relativistically invariant.

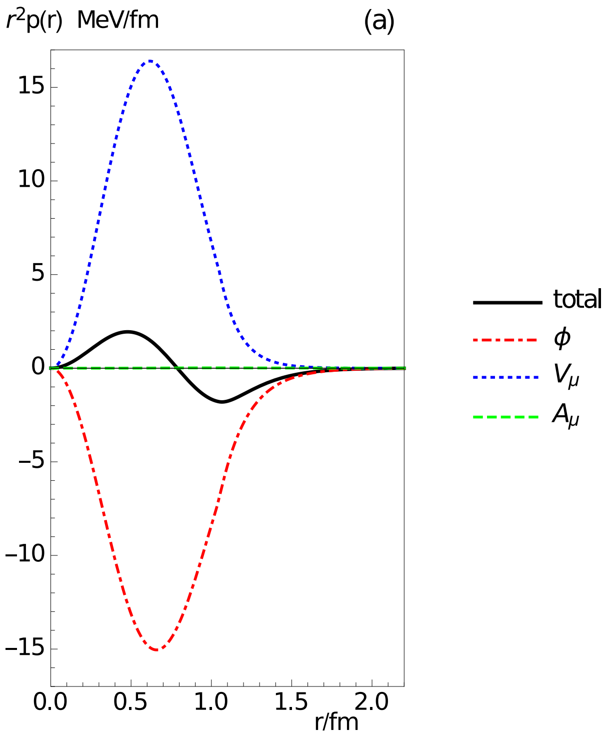
parameters from model QHD-I of the mean field theory of nuclear matter Serot, Walecka (1986)

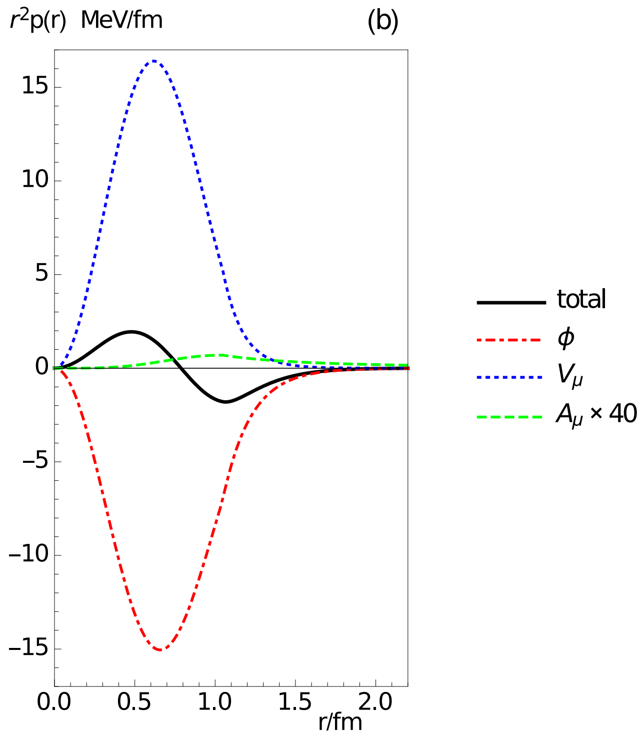
$$m_S=550\,\text{MeV},\quad m_V=783\,\text{MeV},\quad \frac{g_S^2}{4\pi\hbar c}=7.29\,,\quad \frac{g_V^2}{4\pi\hbar c}=10.84\,,\quad \alpha=\frac{e^2}{4\pi\hbar c}=\frac{1}{137},$$

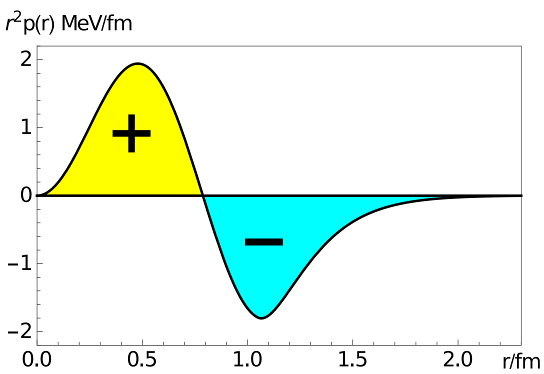
Can be solved analytically, describes particle of charge radius 0.71 fm ("proton") Białynicki-Birula (1993)

We use it to investigate in consistent framework effects of long-range forces Varma, PS (2020)

• **usual features in inner region $r < 2\,\mathrm{fm}$**







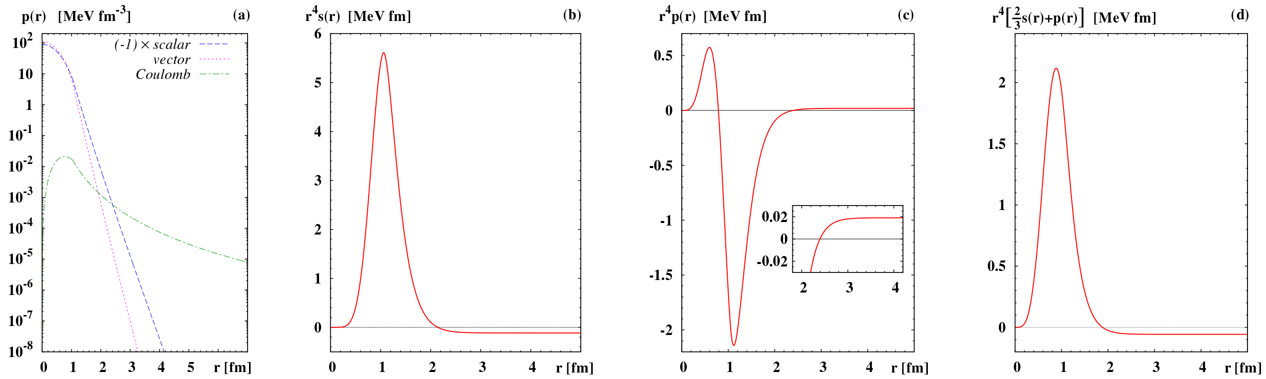
strong forces (scalar and vector fields ϕ and V^μ) make large contributions
about $10\times$ smaller than in chiral quark soliton ("residual nuclear forces")
Coulomb field minuscule contribution, hardly visible

$p(r)$ exhibits node at $r = 0.788\,\mathrm{fm}$, balance of forces:

$$\int dr\; r^2 p_i(r) = \begin{cases} -10.916\,\mathrm{MeV} & \text{for} \quad i = \text{scalar}, \\ +10.891\,\mathrm{MeV} & \text{for} \quad i = \text{vector}, \\ +\;\;0.025\,\mathrm{MeV} & \text{for} \quad i = \text{Coulomb}. \end{cases}$$

So far, same picture as in systems with short-range forces.
But we are looking at the region of $r < 2\,\mathrm{fm}$. Let's look more closely at larger r ...

- **unusual features in outer region $r > 2\,\mathrm{fm}$**



- at large $r > 2$ fm, Coulomb contribution takes over!   Consequences!!

- shear forces $s(r)$ exhibit a node (in short-range systems $s(r) > 0$)
  $p(r)$ has 2$^{\text{nd}}$ node at 2.4 fm (short-range systems one node)
  normal force turns negative (in short range systems $>$ 0)

- model is still mechanically stable:
  dust particles within $R = 1.05$ fm
  where features "as usual"

- D-term is affected by that ...
  (most sensitive to dynamics!!)

• **consequences for D-term**

$$D(t) = (\text{regular strong part}) + \frac{\alpha}{\pi}\Big(-\frac{11}{18} + \frac{\pi^2 M}{4\sqrt{-t}} + \frac{2}{3}\log\frac{(-t)}{M^2}\Big)$$ QED part model-independent!

• from QED diagrams Donoghue, Holstein, Garbrecht, Konstandin,

• long-range tail of distributions $\Leftrightarrow$ small-t behavior of $D(t)$
due to exchange of massless photons (also the "classical Coulomb potential")

• model independent features, seen in
Kubis, Meissner, Nucl. Phys. A **671**, 332 (2000)
Metz, Pasquini, Rodini, PLB **820**, 136501 (2021)
X. Ji and Y. Liu, arXiv:2110.14781 [hep-ph]

Deeper reason:
$T^{ij}(r) = -E^i E^j + \frac{1}{2}\delta^{ij}\vec{E}^{\,2} = -\sigma^{ij}$
(σ^{ij} Maxwell stress tensor, with $\vec{E} \sim \frac{1}{r^2}$ for $r > R$)

$\rightarrow$

$$T_{00}(r)_{\text{QED}} = \frac{1}{2}\frac{\alpha}{4\pi}\frac{\hbar c}{r^4}$$
$$s(r)_{\text{QED}} = -\frac{\alpha}{4\pi}\frac{\hbar c}{r^4}$$
$$p(r)_{\text{QED}} = \frac{1}{6}\frac{\alpha}{4\pi}\frac{\hbar c}{r^4}$$

Important: in classical model **consistently** incorporated!
balance of forces: von Laue condition $\int_0^\infty \mathbf{d}\boldsymbol{r}\,\boldsymbol{r}^2\,\boldsymbol{p}(\boldsymbol{r}) = \mathbf{0}$
consistent nonperturbative solution, proton stable!